

# REPORT DOCUMENTATION PAGE

Form Approved  
OMB No. 0704-0188

Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.

1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE		3. REPORT TYPE AND DATES COVERED	
4. TITLE AND SUBTITLE RESOURCE SCHEDULING FOR THE UNITED STATES ARMY'S BASIC COMBAT TRAINING PROGRAM				5. FUNDING NUMBERS Grant	
6. AUTHOR(S) LTC MICHAEL L. MCGINNIS					
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) OPERATIONS RESEARCH CENTER UNITED STATES MILITARY ACADEMY WEST POINT, NEW YORK 10996				8. PERFORMING ORGANIZATION REPORT NUMBER 95-1	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES) UNITED STATES MILITARY ACADEMY OFFICE OF THE DEAN WEST POINT, NEW YORK 10996				10. SPONSORING/MONITORING AGENCY REPORT NUMBER None	
11. SUPPLEMENTARY NOTES None					
12a. DISTRIBUTION/AVAILABILITY STATEMENT Unlimited				12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words) Each year, the United States Army recruits and trains thousands of new soldiers to fill vacancies in Army organizations created by promotion, transfer or termination of service. Installations responsible for training new soldiers are scattered across the United States. Proper management of the Army's initial entry training program is a very complex, practical military logistics problem that demands timely scheduling of a broad range of reusable training resources. The main objectives of this dissertation are to formulate a mathematical model of the Basic Combat Training phase of initial entry training; formulate an optimal decision process for scheduling training resources based on dynamic programming; formulate a good heuristic procedure for scheduling training resources; incorporate useful performance measures into the formulation of the problem making it possible to discriminate among competing feasible training schedules obtained from heuristic solution methods; and design and implement a fully operational decision support system (DSS) for scheduling basic training resources. DTIC QUALITY INSPECTED 3					
14. SUBJECT TERMS				15. NUMBER OF PAGES 211	
				16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED		18. SECURITY CLASSIFICATION OF THIS PAGE		19. SECURITY CLASSIFICATION OF ABSTRACT	
20. LIMITATION OF ABSTRACT					

DTIC  
ELECTE  
JAN 13 1995  
S G D

19950112 067

NOTICE

NOTICE

NOTICE

DoD Directive 5230.24, "Distribution Statements on Technical Documents," 18 Mar 87, contains seven distribution statements, as described briefly below. Technical Documents must be assigned distribution statements.

DISTRIBUTION STATEMENT A:

APPROVED FOR PUBLIC RELEASE; DISTRIBUTION IS UNLIMITED.

DISTRIBUTION STATEMENT B:

DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES ONLY; (FILL IN REASON); (DATE STATEMENT APPLIED). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (INSERT CONTROLLING DoD OFFICE).

DISTRIBUTION STATEMENT C:

DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND THEIR CONTRACTORS; (FILL IN REASON); (DATE STATEMENT APPLIED). OTHER REQUESTS FOR THIS DOCUMENT SHALL BE REFERRED TO (INSERT CONTROLLING DoD OFFICE).

DISTRIBUTION STATEMENT D:

DISTRIBUTION AUTHORIZED TO DoD AND DoD CONTRACTORS ONLY; (FILL IN REASON); (DATE STATEMENT APPLIED). OTHER REQUESTS SHALL BE REFERRED TO (INSERT CONTROLLING DoD OFFICE).

DISTRIBUTION STATEMENT E:

DISTRIBUTION AUTHORIZED TO DoD COMPONENTS ONLY; (FILL IN REASON); (DATE STATEMENT APPLIED). OTHER REQUESTS SHALL BE REFERRED TO (INSERT CONTROLLING DoD OFFICE).

DISTRIBUTION STATEMENT F:

FURTHER DISTRIBUTION ONLY AS DIRECTED BY (INSERT CONTROLLING DoD OFFICE AND DATE), OR HIGHER DoD AUTHORITY.

DISTRIBUTION STATEMENT X:

DISTRIBUTION AUTHORIZED TO U.S. GOVERNMENT AGENCIES AND PRIVATE INDIVIDUALS OR ENTERPRISES ELIGIBLE TO OBTAIN EXPORT-CONTROLLED TECHNICAL DATA IN ACCORDANCE WITH DoD DIRECTIVE 5230.25 (DATE STATEMENT APPLIED). CONTROLLING DoD OFFICE IS (INSERT).

For further information on withholding of export-controlled, unclassified technical data as referred to in Distribution Statement X, see DoD Directive 5230.25, "Withholding of Unclassified Technical Data From Public Disclosure," issued 6 November 1984.

If you have any questions concerning these distribution statements, please call the Selection Section on (202) 274-7184 or 274-6837 (AUTOVON 284 plus extension).

RESOURCE SCHEDULING FOR THE UNITED STATES ARMY'S  
BASIC COMBAT TRAINING PROGRAM

by

Michael Luther McGinnis

A Dissertation Submitted to the Faculty of the  
DEPARTMENT OF SYSTEMS AND INDUSTRIAL ENGINEERING

In Partial Fulfillment of the Requirements  
For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

1994

Accession For	
NTIS CRA&I	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution /	
Availability Codes	
Dist	Avail and/or Special
A-1	

THE UNIVERSITY OF ARIZONA  
GRADUATE COLLEGE

As members of the Final Examination Committee, we certify that we have  
read the dissertation prepared by Michael Luther McGinnis  
entitled RESOURCE SCHEDULING FOR THE UNITED STATES ARMY'S

BASIC COMBAT TRAINING PROGRAM

and recommend that it be accepted as fulfilling the dissertation  
requirement for the Degree of DOCTOR OF PHILOSOPHY

Emmanuel Fernandez  
Dr. Emmanuel Fernandez

6-13-74  
Date

Pitu Mirchandani  
Dr. Pitu Mirchandani

6-15-74  
Date

Ferenc Szidarovszky  
Dr. Ferenc Szidarovszky

6-15-74  
Date

          
Date

          
Date

Final approval and acceptance of this dissertation is contingent upon  
the candidate's submission of the final copy of the dissertation to the  
Graduate College.

I hereby certify that I have read this dissertation prepared under my  
direction and recommend that it be accepted as fulfilling the dissertation  
requirement.

Emmanuel Fernandez  
Dissertation Director

6-15-74  
Date

Dr. Emmanuel Fernandez

Dr. Pitu Mirchandani



## STATEMENT BY AUTHOR

This dissertation has been submitted in partial fulfillment of requirements for an advanced degree at The University of Arizona and is deposited in the University Library to be made available to borrowers under rules of the Library.

Brief quotations from this dissertation are allowable without special permission, provided that accurate acknowledgment of source is made. Requests for permission for extended quotation from or reproduction of this manuscript in whole or in part may be granted by the head of the major department or the Dean of the Graduate College when in his or her judgment the proposed use of the material is in the interests of scholarship. In all other instances, however, permission must be obtained from the author.

SIGNED: \_\_\_\_\_

## ACKNOWLEDGMENTS

I wish to express my gratitude to my coadvisors, Professor Emmanuel Fernández-Gaucherand and Professor Pitu B. Mirchandani (The University of Arizona) for leading me toward completion of this dissertation. Without their patient guidance and continuous encouragement, this work could never have been completed. Their informative reviews of this dissertation made it more understandable and complete by correcting errors and omissions.

Appreciation is extended to Professor Ferenc Szidarovszky of the Systems and Industrial Engineering Department, and to Professor Douglas Vogel and Professor Pamela Slaten of the Management Information Systems Department (University of Arizona) for their support as members of my doctoral committee.

Special thanks to Professor Jeff Goldberg (University of Arizona) for helping with the integer programming formulations of the basic training problem.

Thanks to Joseph Potter for his efforts to help me with implementing the computer code for the dynamic programming method, and to Captain Edward Pohl (USAF) for listening when I needed to think out loud.

Special thanks to Colonel James L. Kays (United States Military Academy) for making this opportunity possible, and for sound advice, unfailing support and words of encouragement when I needed them.

Thanks to then Brigadier General Theodore Stroup, then Deputy Chief of Staff for Resource Management of the U.S. Army Training and Doctrine Command (TRADOC) Headquarters, Fort Monroe, VA, for sponsoring my work on the training base problem in 1988 and 1989.

Thanks also to then Lieutenant Colonel David Hardin, then of the Planning, Programming, Analysis and Evaluation Directorate, and thanks to Mr. Rich Wagner of the Training and Operations Management Directorate, TRADOC Headquarters, Fort Monroe, VA for their willingness to share their insights into the dynamics of the initial entry training process.

This dissertation is dedicated to my family: my wife Tracy Ann, my children, Meghan, Matt, and Meredith, and my parents, Jim and Bonnie for their love, understanding, and inspiration.

## TABLE OF CONTENTS

LIST OF FIGURES . . . . .	7
LIST OF TABLES . . . . .	9
ABSTRACT . . . . .	10
1. INTRODUCTION . . . . .	12
2. MODEL FORMULATION . . . . .	18
2.1 Dynamics of the Basic Training Problem . . . . .	18
2.2 Related Work . . . . .	23
2.3 Mathematical Formulation of the Basic Training Problem . . . . .	26
3. OPTIMAL DECISIONS USING DYNAMIC PROGRAMMING . . . . .	33
3.1 Complexity of the Basic Training Optimal Decision Problem . . . . .	33
3.2 The Basic Training Optimal Decision Problem . . . . .	36
3.3 State Augmentation . . . . .	45
3.4 DP Model of the Basic Training Problem . . . . .	51
3.5 Complexity of Solving the Augmented State Space Problem . . . . .	53
4. HEURISTIC APPROACHES . . . . .	55
4.1 Overview of the Heuristic Approaches . . . . .	55
4.2 Single-Pass Heuristic (SPH) . . . . .	58
4.3 Multi-Pass Heuristic (MPH) . . . . .	69
5. DECISION SUPPORT SYSTEM (DSS) . . . . .	73
5.1 Preliminary Work . . . . .	75
5.2 System Development Process . . . . .	78
5.3 System Architecture . . . . .	81
5.4 Resource Costing Module . . . . .	83
5.5 System Benefits . . . . .	89

6. RESULTS . . . . .	91
6.1 Heuristic Scheduling Results . . . . .	92
6.2 Comparison of Optimal and Heuristic Scheduling Results . . . . .	103
7. CONCLUSIONS . . . . .	119
7.1 Summary and Contributions of Work . . . . .	119
7.2 Suggestions for Future Research . . . . .	122
APPENDIX A: SUMMARY OF THE LITERATURE SEARCH . . . . .	124
APPENDIX B: TRAINING BASE SCENARIO DECISION PROCESS. . . . .	128
APPENDIX C: ILLUSTRATIVE SESSION WITH THE . . . . . DECISION SUPPORT SYSTEM	129
APPENDIX D: INDIRECT TRAINING PROGRAM COSTS . . . . .	141
APPENDIX E: COST FACTORS . . . . .	144
APPENDIX F: ADDITIONAL TRAINING PROGRAM COST ESTIMATES	152
APPENDIX G: IMPLEMENTATION OF DYNAMIC PROGRAMMING FOR THE ONE-PERIOD LAG PROBLEM	155
REFERENCES . . . . .	210

## LIST OF FIGURES

FIGURE 1. Initial Entry Army Training Installations (1988)	12
FIGURE 2. Aggregated View of the Initial Entry Training Process	13
FIGURE 3. Disaggregated View of the Initial Entry Training Process (1988)	14
FIGURE 4. Percent of Annual Recruit Arrivals by Month	19
FIGURE 5. <i>Phase I</i> Initial Resource Scheduling Algorithm	60
FIGURE 6. <i>Phase I</i> Single-Pass Heuristic Policy Improvement Algorithm	61
FIGURE 7. <i>Phase I</i> Cycle Length Policy Adjustment	62
FIGURE 8. <i>Phase II</i> Single-Pass Heuristic Policy Improvement Algorithm	67
FIGURE 9. <i>Phase III</i> Multi-Pass Heuristic Policy Improvement Algorithm	70
FIGURE 10. Decision Support System Development Phases	80
FIGURE 11. Decision Support System Architecture and System Modules	82
FIGURE 12. Categories of Training Resource Costs	85
FIGURE 13. Comparison of Heuristic Processing Times	96
FIGURE 14. Comparison of Instructor-to-Student Ratios	97
FIGURE 15. Effectiveness of Heuristic Solutions as a Percent of the <i>Utopian</i>	99
FIGURE 16. Comparison of Idle Training Companies	100
FIGURE 17. Comparison of Total Training Program Cost by Scenario	103
FIGURE 18. Comparison of DP Versus Heuristic Solutions to the One-Period Lag Problem	113
FIGURE 19. Comparison of Company Strengths for Example 1	115
FIGURE 20. Comparison of Company Strengths for Example 3	116
FIGURE 21. Comparison of Idle Training Companies for Example 1	117
FIGURE 22. Comparison of Idle Training Companies for Example 3	118
FIGURE 23. Training Base Scenario Decision Process	128

FIGURE 24. Numerical Output from the Decision Support System . for Scenario 11	130
FIGURE 25. Training Company Shortfalls Before Heuristic Scheduling .	136
FIGURE 26. Training Company Shortfalls After the Resource Scheduling . Algorithm of <i>Phase I</i>	137
FIGURE 27. Results Showing the Initial Feasible Training Schedule of <i>Phase I</i>	138
FIGURE 28. Training Company Shortfalls After Deactivating Training Companies in <i>Phase II</i>	139
FIGURE 29. Training Company Shortfalls After Company Strength Policy Improvement Algorithm via Multi-Pass Heuristic	140

## LIST OF TABLES

TABLE 1.	Training Cost Factors for Basic Training Analysis	.	.	86
TABLE 2.	Training Company Deactivation Scenarios	.	.	92
TABLE 3.	Initial Company Strength Values	.	.	92
TABLE 4.	Numerical Results of Scheduling Heuristics	.	.	95
TABLE 5.	Summary of Training Resource Costs	.	.	102
TABLE 6.	Increase in the Size of the State Space for each One Period Increase in the Time Lag of the Dynamic Model	.	.	106
TABLE 7.	Scheduling Results for the One-Period Lag Problem	.	.	112
TABLE 8.	Literature Search Results	.	.	125
TABLE 9.	Summary of 1989 Costs for Scenario 11	.	.	153
TABLE 10.	Summary of 1990 Costs for Scenario 11	.	.	153
TABLE 11.	Comparison of Annual Costs for Scenario 11	.	.	154

## ABSTRACT

Each year, the United States Army recruits and trains thousands of new soldiers to fill vacancies in Army organizations created by promotion, transfer, or termination of service. Installations responsible for training new recruits are scattered across the United States. Initial entry training for new recruits is conducted in two phases: Basic Combat Training followed by Advanced Individual Training.

Proper management of the Army's initial entry training program is a very complex, practical military logistics problem that demands timely scheduling of a broad range of reusable training resources, such as, training companies. Currently, manual heuristic methods are used to schedule training companies throughout the planning horizon to support initial entry training, where training company scheduling also involves deciding how many recruits to assign to training companies each week. These methods have evolved over a number of years when there were few changes to the training base, and recruiting levels remained relatively stationary. Unfortunately, there are several severe shortcomings with these methods. For example, determining the number of recruits assigned per training company and the number of weeks a training company remains busy training recruits is a manual trial-and-error process. Second, it is possible for different analysts to generate different solutions for the same recruitment scenario. Third, no methods exist for conducting comparative analyses to appraise the quality of competing feasible training schedules. Finally, the temporal interdependence of decisions makes decision variables in the future periods depend on current decision variables. This complicates resource scheduling and makes the manual generation of week-by-week training schedules a tedious, time-consuming task.

This dissertation: (1) formulates a mathematical dynamic model of the Basic Combat Training phase of initial entry training; (2) formulates a decision model for



optimally scheduling training resources based on dynamic programming; (3) formulates an improved heuristic procedure for scheduling training resources; (4) incorporates a "training quality" performance measure into the formulation of the objective function making it possible to compare competing feasible training schedules obtained by various methods; and (5) designs, develops and implements a fully operational computer-based decision support system (DSS) for scheduling basic training resources.

The computational experiments reveal that the heuristic procedures developed are indeed computationally efficient and provide "good" solutions in terms of training "quality," resource utilization, and training cost.

## 1. INTRODUCTION

Each year, the United States Army recruits and trains thousands of new soldiers to fill vacancies in Army organizations created by promotion, transfer, or termination of service. Responsibility for initial entry training belongs to the U.S. Army Training and Doctrine Command (TRADOC) headquartered at Fort Monroe, Virginia. The installations responsible for training new recruits are scattered across the United States as shown in Figure 1.

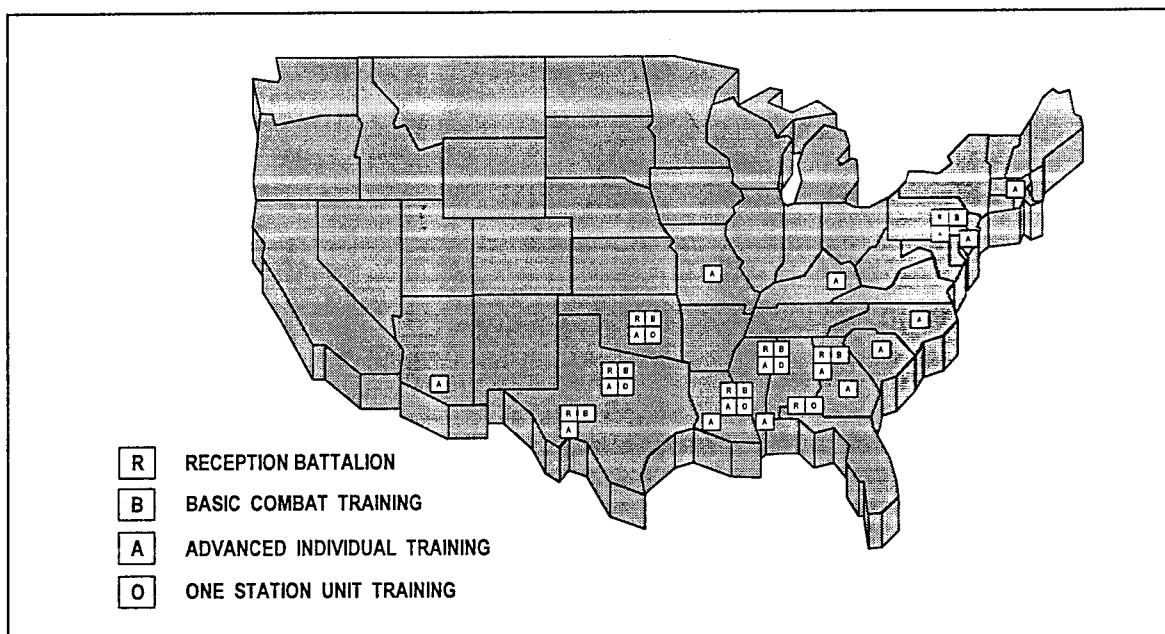


Figure 1. Initial Entry Army Training Installations (1988)

As new recruits arrive at initial entry training installations, they are temporarily assigned to a reception battalion where they are in-processed before beginning their initial entry training. Ideally, the amount of time recruits are assigned to the reception battalion does not exceed one week.

Entry level training consists of two phases: Basic Combat Training (BCT), normally lasting eight weeks, followed by Advanced Individual Training (AIT) which

varies from five to fifty weeks. The variability in time of AIT reflects curriculum differences across the AIT programs administered at different installations. In One-Station-Unit Training (OSUT), BCT and AIT take place at the same installation. Figure 2 illustrates an *aggregated* view of the initial entry training process where, for each training phase, the training companies from all installations can be viewed as a single group of reusable resources, and an important aspect of scheduling is determining how many recruits to assign to training companies each week.

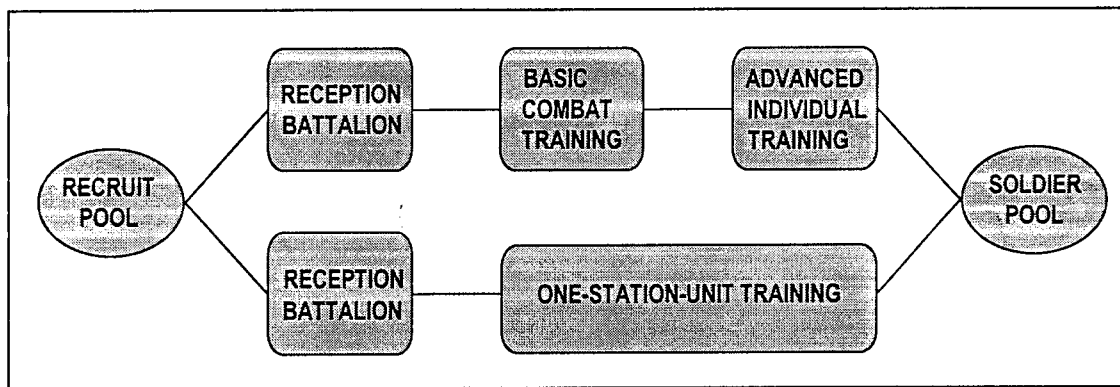


Figure 2. Aggregated View of the Initial Entry Training Process

Figure 3 breaks down, or *disaggregates*, the training process by training phase and training installation. The arcs connecting BCT and AIT training installations in Figure 3 represent the flow of recruits who complete Basic Combat Training and continue on with Advanced Individual Training.

Proper management of the military's initial entry training program is a very complex task that requires timely scheduling of a broad range of *reusable* training resources needed for training new recruits. Modeling the initial entry training process is complicated by the combinatorial complexity of (1) the decision space and (2) the state space needed for computing optimal scheduling decisions (see Section 3.1).

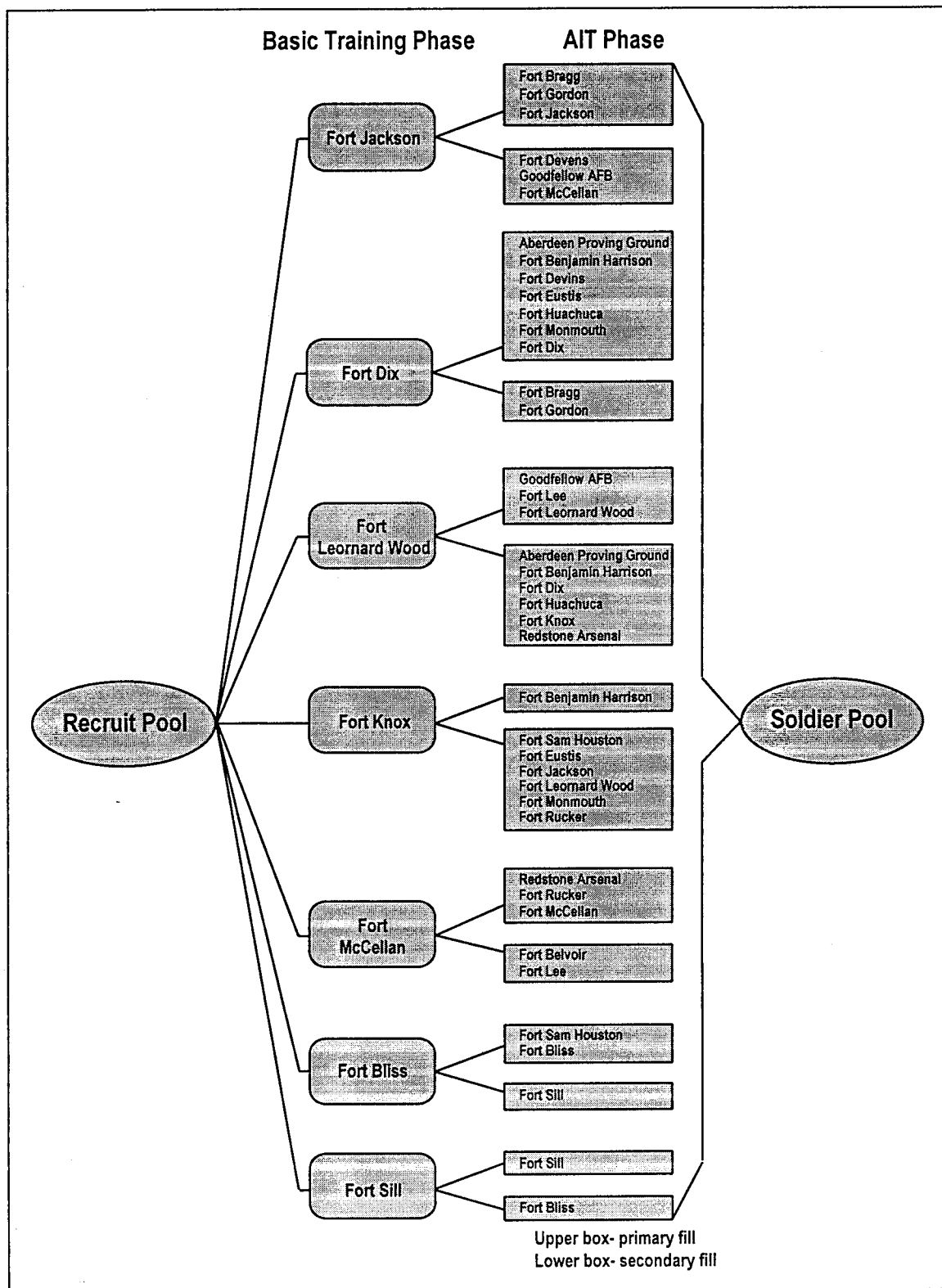


Figure 3. Disaggregated View of the Initial Entry Training Process (1988)

In practice, reusable training resources are being continuously rescheduled to support training over a rolling planning horizon. Although numerous types of reusable training resources may be scheduled to support initial entry training, the scheduling problem presented here focuses on scheduling only one resource for one training phase; namely, scheduling *basic training companies* for the aggregated Basic Combat Training phase.

Present methods for scheduling training companies to support Basic Combat Training rely on heuristics that have evolved over a number of years when recruiting targets have remained relatively stable and there have been only a few changes to the training installation complex. Unfortunately, severe shortcomings exist with these scheduling methods, henceforth referred to as *heuristics-used-in-practice* (HUIP). For example, determining the number of recruits assigned per training company (*company strength*) and the number of weeks the training companies remain busy training a particular group of new recruits (*training cycle length*) is essentially a manual trial-and-error process. Second, it is possible to generate different schedules for allocating training resources for the same recruitment (i.e., training) scenario. Third, no methods exist for making comparative analyses to appraise the quality of competing feasible schedules for training resource allocations. Finally, the interdependence of the problem's decision variables (e.g., company strengths and training cycle lengths) causes decisions made for the current period to impact future decision epochs (see Section 3.3, *State Augmentation*, for details). This complicates the decision process and makes generating week-by-week training schedules a tedious, time-consuming task.

The summary of the dissertation is as follows.

Chapter 2 presents mathematical notation and a mathematical model of the Basic Combat Training phase of initial entry training; henceforth referred to as the *basic*

*training problem.* Salient dynamics of Basic Combat Training essential to the development of the mathematical model of the basic training problem are discussed.

Chapter 3 formulates an optimal decision model for scheduling training resources based on dynamic programming [3]. The value of any training schedule may be measured by numerous performance measures, such as, training quality, training costs, or the length of time required to execute the training program, to name a few. For this study, the basic training problem is formulated using a "training quality" performance measure called the *instructor-to-student ratio* (see Section 2.1). An alternate objective that minimizes basic training costs is also presented.

Chapter 4 develops an efficient heuristic scheduling method for the basic training problem that generates "good" training resource schedules in a timely manner. The heuristic decision model is motivated by the policy iteration step of dynamic programming's policy improvement algorithm. The instructor-to-student ratio is also used in the heuristic decision model to discriminate among competing feasible training resource schedules, thereby correcting one of the shortfalls of the heuristics-used-in-practice (HUIP) described above.

Chapter 5 discusses the development and implementation of a fully operational computer-based decision support system (DSS) capable of supporting analyses of a broad range of problems related to initial entry training that features fully automated heuristic scheduling procedures. Automation of the heuristic scheduling process corrects another one of the major shortcomings of scheduling with heuristics-used-in-practice.

Chapter 6 gives results that compare the quality of heuristic scheduling methods for various performance measures using a set of twelve real-world test scenarios. Section 6.2 discusses implementation of the DP algorithm and gives results comparing the quality

of optimal versus heuristic scheduling methods for one small problem using a different set of twelve test scenarios.

Chapter 7 summarizes the work, discusses the contribution of the dissertation and presents some concluding remarks that include suggestions for future research.

## 2. MODEL FORMULATION

Before presenting the mathematical model of the basic training problem, practical aspects of Basic Combat Training essential to the proper development of the dynamic training system model are discussed.

### 2.1 Dynamics of the Basic Training Problem

#### *Estimating the Weekly Arrival of New Recruits*

In the basic training problem, demand for training resources depends upon the number of new recruits who report for training each week. In the real-world problem, the arrival of recruits is a random parameter which makes the demand for training resources a random variable. Under these circumstances, demand would be specified by a probability distribution. However, for the version of the basic training problem presented here, recruit arrivals are estimated ahead of time for each week  $t$  and year  $j$  of the planning horizon  $T_j$ , given an annual recruiting target for each year  $j$ . Therefore, the absence of a random disturbance makes this formulation of the basic training problem completely deterministic (see *Demand* of Section 3.2 for additional details).

Recruiting continues throughout the year and focuses heavily on young people in their final year of high school. However, high school seniors cannot be scheduled for Basic Combat Training until after graduation which normally occurs in late spring or early summer. This causes recruit arrivals to surge during summer months (*surge period*) with fewer arrivals during the rest of the year (*nonsurge period*). Surge periods also occur around Thanksgiving and Christmas due to a break in training during the holiday period (see Figure 4).



Historical recruit training data for this study was provided by the U.S. Army Training and Doctrine Command Headquarters<sup>1</sup> in the form of monthly summary reports for the years 1984 through 1987. Figure 4 shows the historical percent of annual recruit arrivals by month.

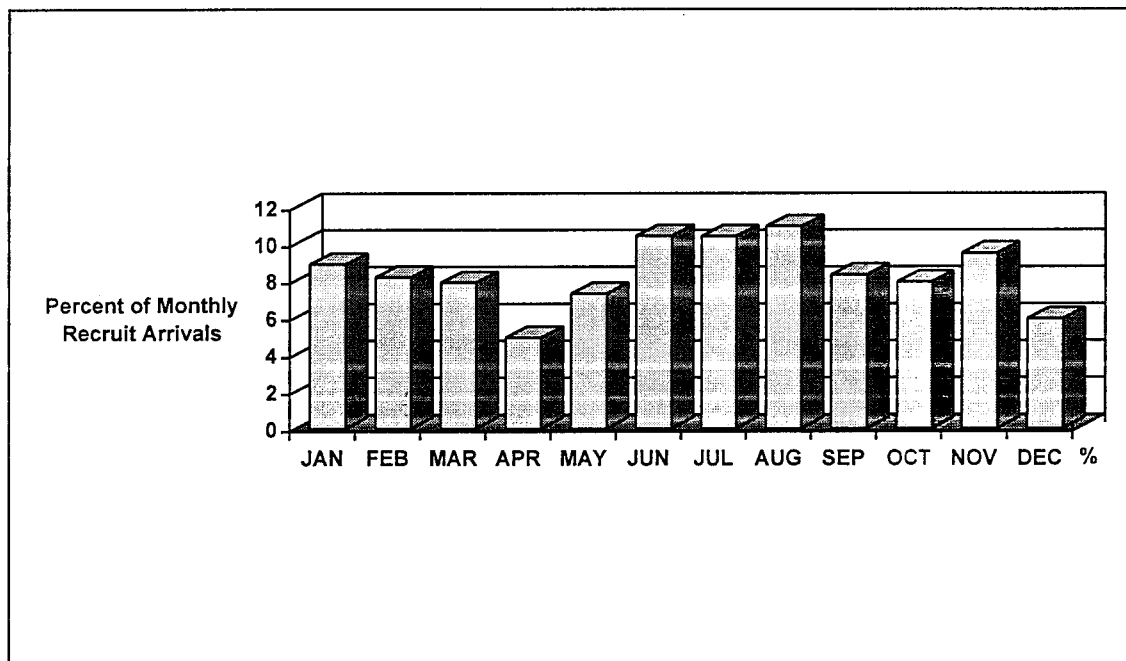


Figure 4. Percent of Annual Recruit Arrivals by Month

The monthly data from Figure 4 has been interpolated into a weekly distribution of recruit arrivals over the twelve months of the year. The week-by-week arrival of recruits is estimated by multiplying the annual recruiting target, times the weekly relative frequency distribution of arrivals, times the rate at which new recruits report for Basic Combat Training (*show rate*). Analysis of the historical data reveals that although annual recruiting targets vary from year-to-year, the distribution of recruit arrivals across the year remains relatively stationary. As a practical matter, we recommend that the

<sup>1</sup>Courtesy of then Lieutenant Colonel David Hardin of the Planning, Programming, Analysis and Evaluation Directorate of Fort Monroe, VA.

distribution of recruit arrivals be updated at the beginning of each year using recruit arrival data from the previous year. This is done so that the "current" frequency distribution of recruit arrivals reflects potential changes in recruiting trends that may affect the training requirement.

### *Dynamics of Varying Training Company Strength*

Company strength, the number of recruits assigned per training company, is bounded below at 150 and above at 250 recruits. In practice, companies scheduled to start training in the same week are initially assigned equal strengths. This rule simplifies logistical problems associated with managing the Basic Combat Training program and is incorporated into our basic training model. Determining the company strength for a given week  $t$  requires the following information:

1. the number of recruits that report for training in week  $t$ ; and
2. the number of training companies available at the beginning of week  $t$  to start training new recruits.

However, the number of training companies available to start training in week  $t$  depends upon past company strength decisions (see Section 3.3, *State Augmentation*). For example, if company strength decisions made prior to week  $t$  resulted in company strengths near the lower bound, then more companies started training in those weeks than would have started if company strengths had been nearer the upper bound. This leaves fewer companies available to start training recruits in week  $t$ . In any week, it is possible for previous weeks' company strength decisions to result in a *training company shortfall*, where the number of training companies is not sufficient to handle the arrival of new

recruits (or *training load*). In such cases, the company strength decisions for previous weeks must be sequentially revised (if possible) to correct the training company shortfall.

### *Instructor-to-Student Ratio*

Basic training is, by design, highly stressful for new recruits; an aspect of training that often has a negative effect on recruit learning and retention. Training managers rely on quality instruction and close supervision of recruits to offset some effects of stress. In practice, training managers use the ratio of instructors-to-students as a guide when manually sizing training companies with the heuristics-used-in-practice. Training managers attempt to keep the instructor-to-student ratio around 1-to-16 (or lower if possible) resulting in company strengths of approximately 200 recruits per company.

The instructor-to-student ratio serves as the primary objective in the formulation of the optimal decision model presented in Chapter 3. The automated heuristic scheduling methods developed in Chapter 4 also make use of the instructor-to-student ratio, both, as a rule for establishing company strengths, and as a performance measure for comparing suboptimal solutions.

### *Compressing-the-Load*

Each week, recruits who report to basic training installations are assigned to training companies. This *fill week* runs from Saturday through midnight Thursday. Basic Combat Training begins on Friday and, in general, lasts eight weeks. Recruits who graduate from BCT continue with Advanced Individual Training. Normally, a *maintenance week* is scheduled following BCT to repair equipment and facilities before the next training cycle begins. This ten-week sequence is called a *normal training cycle*.

In some cases, it may not be possible to eliminate a training company shortfall by adjusting company strengths alone (see above). Another way to correct a training

company shortfall is to make more companies available in week  $t$  by shortening the training cycles for companies that started either eight or nine weeks earlier. This is done by eliminating either the fill week or the maintenance week, or both, from those company's (normal) training cycles. However, this practice, called *compressing-the-load*, can have a negative impact on training company cadre. Eight weeks of basic training are, in many ways, as stressful for instructors as for recruits. Cadre typically spend 15 hours per day training recruits which leaves little time for personal business. Reducing or eliminating the break between training cycles can decrease cadre effectiveness which works against the goal of quality instruction. Therefore, compressing-the-load is only used when absolutely necessary (e.g., when the demand for training companies cannot be met by adjusting company strengths).

### *Backlogging Recruits*

Occasionally, the number of recruits who report for basic training in a given week exceeds the number of training spaces available, meaning there are not enough training companies available to enable all recruits to begin basic training that week (i.e., a training company shortfall occurs). In those instances, some recruits must be held back until the next week to start training. However, training experts observe that recruits who are held back, or *backlogged*, tend to have motivational or behavioral problems leading to poor performance, or worse, causing them to fail basic training. As a result, training managers avoid backlogging recruits whenever possible and it is not included in model formulation at this time.

## 2.2 Related Work

A thorough literature survey (see Appendix A) failed to surface any papers directly related to the basic training problem. However, the literature review did locate papers on related resource scheduling problems that generated ideas for attacking the basic training problem.

For example, Yang and Ignizio [13] solve a (somewhat) related military resource scheduling problem in two phases using two different heuristics; an approach that is (coincidentally) similar to the three-phase, two-heuristic method presented in Chapter 4 for solving the basic training problem. However, Yang and Ignizio deal with the problem of scheduling daily training activities for a fixed number of Army battalions<sup>2</sup> located at the same installation where (1) the type and quantity of resources to support training are constrained, (2) precedence relationships exist between tasks, and, in some cases, (3) two or more battalions must work together to accomplish training tasks and must share resources as well. The problem described by Yang and Ignizio contains tens of millions decision variables; they consider their problem to be NP-complete and beyond the scope of exact solution methods. Therefore, Yang and Ignizio propose a heuristic method to determine suboptimal training activity schedules (by battalion) that minimize the total time required for the battalion(s) to complete daily training tasks for a finite planning horizon. Training activity and resource scheduling is accomplished in two phases. In phase one, a "greedy" scheduling algorithm makes a single pass through the planning horizon to obtain an initial feasible schedule that is free of scheduling conflicts (e.g., no violations of task precedence and no unresolved demands for training resources). Phase

---

<sup>2</sup>An Army battalion, in general, consists of five companies; a headquarters company, a service support company and three "line" companies. Basic training battalions generally consist of a headquarters company and five basic training companies.

two improves the initial schedule using a "search and reshuffle" exchange heuristic that reduces the number of days required to execute the training program.

Another related problem, first introduced in 1958 by Manne [9] and Wagner and Whitin [12], is the economic lot-sizing problem (ELSP) for material requirements planning (MRP) in manufacturing and inventory processes. See Afentakis [1], Afentakis et al. [2], Billington et al. [4], Blackburn and Millen [5], Crowston et al. [6], Graves [8] and Zangwill [14] for extensions to the basic ELSP and different methods for solving the problem. In the economic lot-sizing problem, items are produced in batches (lots) to meet demand in the current and future periods. As demand depletes inventory, more of the item must be periodically produced to prevent a stock-out. In the Wagner and Whitin version, a single item is produced to meet known demand for each period of a finite planning horizon, where the objective is to minimize the total cost of setting up for production, production, and inventory holding. Problem constraints include no backorders (i.e., a nonnegative production constraint signifying no backlogging of production to satisfy unmet demand), and zero inventory on-hand at the start and end of the planning horizon. A balance equation accounts for inventory carried forward to meet future demand. Wagner and Whitin solved their problem using dynamic programming (DP). The DP algorithm makes one pass through the planning horizon to determine an optimal production policy that minimizes total cost. The amount to produce in each period is fixed, so the solution is completely specified by the sequence of productions (e.g., the production policy) that determine whether or not to set-up and produce in each period.

In some ways, the basic training problem is similar to the basic economic lot-sizing problem. The aggregated view of initial entry training in Figure 2 is equivalent to a two-phase (BCT-AIT) production process where the training companies in each phase

are treated as a single group of parallel servers. The "output" of the first phase (BCT) is a single item (BCT graduates). In the BCT phase, both training company strengths (batch sizes) and training cycle lengths (production lead times) vary from period-to-period, but are constrained (see *Modeling Constraints* in Section 2.3). The second phase (AIT) is a multi-item problem, where each AIT program produces a different type of "product" (AIT graduate), and both company strengths and training cycle lengths may vary across installations, and within an installation as well.

Despite the similarities, there are major differences between the two problems which prevent ELSP solution methods from being applied directly to the basic training problem. For example, the objective of the economic lot-sizing problem is to find a production schedule that meets demand by minimizing production costs under certain assumptions and constraints. However, the objective currently formulated for the basic training problem seeks to maximize the "quality" of training in each period of the planning horizon (see *Instructor-to-Student Ratio* above).

*Demand* also discriminates between the two problems. In the ELSP, demand is measured by external demand for the item being produced. An equivalent demand in the basic training problem would be demand for recruits by military commanders to fill vacancies in their units. However, demand in the basic training problem is actually measured by the requirement for training resources as determined by the time-varying (seasonal) arrival of new recruits to start basic training each period.

Finally, the decision elements of the two problems are different. In the Wagner and Whitin version of the ELSP, the problem is to determine whether or not to produce a fixed amount of the item in each period to meet demand. However, there are three interdependent decision elements in the basic training problem that can take on a range of values in each period. The decision elements are:

1. company strengths that vary from 150 to 250 recruits per company;
2. training cycle lengths that are generally ten weeks but may be shortened by one or two weeks to correct a training company shortfall; and
3. the number of training companies lost (*deactivated*) or gained (if any) in each period representing a change to the training base structure.

### 2.3 Mathematical Formulation of the Basic Training Problem

Mathematical notation for the problem is as follows.

$j$ : year of the planning horizon,  $j \in \{1, 2, \dots, J\}$ ;

$t$ : week of a given year  $j$ ,  $t \in \{1, 2, \dots, T_j\}$  where  $T_j$  is the number of weeks in year  $j$ ;

$\delta(t)$ : recruit show rate for week  $t$  where  $0 \leq \delta(t) \leq 1$ ;

$p(t)$ : relative frequency distribution of recruit arrivals over week  $t$  of any year;

$D_j$ : total number of training companies deactivated in year  $j$ ;

$M_j$ : number of training companies available at the beginning of year  $j$ ;

$R_j$ : recruiting objective for year  $j$  determined by Department of the Army;

$d_j(t)$ : number of training companies to deactivate in week  $t$  of year  $j$ ;

$r_j(t)$ : estimated number of recruits that show up for training in week  $t$  of year  $j$ ;

$x_j(t)$ : strength of training companies starting in week  $t$  of year  $j$ ;

$\bar{X}$ : upper bound for training company strength;

$\underline{x}$ : lower bound for training company strength;

$y_j(t)$ : training cycle length for companies starting in week  $t$  of year  $j$ ;

$\bar{Y}$ : upper bound for training cycle length;

$\underline{y}$ : lower bound for training cycle length;



$D_j^*(t)$ : balance of training companies left to be deactivated as of week  $t$  of year  $j$ ;

$I_j(t)$ : number of idle training companies at the beginning of week  $t$  of year  $j$ ;

$\bar{I}$ : upper bound for the idle training company constraint.

### *Modeling Assumptions*

The major modeling assumptions included in the formulation of the basic training problem are:

- finite planning horizon of  $T_j$  equal periods;
- varying but bounded training company strengths;
- varying but bounded training cycle lengths;
- no backlogging of recruits, or equivalently, no backlogging of the requirement for training companies;
- changes, if any, to the number of training companies available for training are constrained to decreases (*deactivations*) in training companies.

### *Modeling Constraints*

$$\underline{x} \leq x_j(t) \leq \bar{X}: \quad \text{company strength constraint;} \quad (1)$$

$$\underline{y} \leq y_j(t) \leq \bar{Y}: \quad \text{training cycle constraint;} \quad (2)$$

$$M_j \geq D_j: \quad \text{deactivation scenario constraint;} \quad (3)$$

$$d_j(t) \geq 0 \quad \forall (t,j): \quad \text{company deactivation constraint;} \quad (4)$$

$$0 \leq I_j(t) \leq \bar{I} \quad \forall (t,j): \quad \text{problem feasibility constraint.} \quad (5)$$

### *Modeling Relationships and Equations*

The expected number of new recruits to arrive for training in week  $t$  of year  $j$  is

$$r_j(t) = \delta(t) p(t) R_j, \quad (6)$$

where the recruiting objective  $R_j$ , determined by the Department of the Army (DA), is greater than the number of new soldiers required to meet the needs of the Army in a given year  $j$ .

The number of training companies that remain to be deactivated as of week  $t$  of year  $j$  is determined according to

$$D_j^*(t) = D_j - \sum_{i=1}^t d_j(i), \quad (7)$$

$$\text{where } D_j = \sum_{t=1}^{T_j} d_j(t). \quad (8)$$

The expected number of companies to begin training in week  $t$  of year  $j$  is given by

$$\frac{r_j(t)}{x_j(t)}, \quad (9)$$

where it is important to note that  $x_j(t)$  is the company strength decision to take in week  $t$ .

The number of companies to become available in week  $t$  having just completed a training cycle that began  $l \in L$  weeks earlier is

$$\sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)}. \quad (10)$$

The possible values for  $l$  in any week  $t$  are within the set  $L \in \{ (10), (10,9), (10,9,8) \}$ , where

$L = (10) \equiv L_{MIN}$  indicates only one group of training companies becomes available in week  $t$  of year  $j$ ; the group that just completed a normal 10-week training cycle.

$L = (10,9)$  indicates two groups of training companies become available in week  $t$  of year  $j$ ; those finishing 10-week and 9-week training cycles. The group of training companies finishing a 9-week training cycle had its cycle compressed by one week (see Section 2.1 for an explanation of *compressing-the-load*).

$L = (10,9,8) \equiv L_{MAX}$  indicates three groups of training companies become available in week  $t$  of year  $j$ ; those finishing 10-week, 9-week, and 8-week cycles. Therefore, the training cycles for training companies completing 9- and 8-week training cycles were compressed by one and two weeks, respectively.

A balance equation accounts for the number of idle companies carried forward to week  $t+1$  of year  $j$  to meet demand for training companies. An idle training company is one that is available but not required to start training at the beginning of a week. The balance equation for idle training companies is given by

$$I_j(t+1) = I_j(t) + \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} - \frac{r_j(t)}{x_j(t)} - d_j(t). \quad (11)$$

The number of idle companies at the end of a week is always carried forward, available to meet the training requirement in the next week.

Although training companies is integer-valued, computing (11) may generate a non-integer result due to computing (9) and (10). To overcome this problem, (9) and (10) are rounded down when  $x_j(t) < \bar{X}$ , and are rounded up to the next whole training company when  $x_j(t) = \bar{X}$ . This creates two cases for computing (11):

Case 1:  $x_j(*) < \bar{X}$ ,

$$I_j(t+1) = I_j(t) + \left\lfloor \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} \right\rfloor - \left\lfloor \frac{r_j(t)}{x_j(t)} \right\rfloor - d_j(t); \quad (12)$$

Case 2:  $x_j(*) = \bar{X}$ ,

$$I_j(t+1) = I_j(t) + \left\lceil \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} \right\rceil - \left\lceil \frac{r_j(t)}{x_j(t)} \right\rceil - d_j(t). \quad (13)$$

When the current company strength constraint is *tight* (i.e., an equality constraint) at the upper bound, then the fractional part of the training company is rounded up, as denoted by the ceiling operator  $\lceil * \rceil$ . When training companies are at full strength, those recruits that are represented by the fractional part of a training company can only begin basic combat training if an additional training company is scheduled to start. In all other cases, the fractional part may be dropped, as denoted by the floor operator  $\lfloor * \rfloor$ , since, in general, sufficient training spaces will be available in training companies not filled to

capacity to absorb the recruits represented by the fractional part of a training company. For simplicity, the floor and ceiling operators of (12) and (13), respectively, (denoting the rules for rounding training companies) will not be repeated for every future reference to training company computations, such as in (11). However, it is to be understood that these rules are in effect throughout the dissertation unless stated otherwise.

The rounding rules given here for estimating the number of training companies required to train recruits each week are consistent with the rounding rules used in practice. However, the practice of "redistributing" the number of recruits represented by the "dropped" fractional part of a training company when  $x_j(t) < \bar{X}$  fails to account for the true company strength which is larger than the prescribed  $x_j(t)$ . This leads to inaccuracies in the estimates of (1) training "quality" (as measured by the instructor-to-student ratio; see Section 2.1) and (2) training program costs (based on variable costs per trainee). Therefore, even though we do not expect radical changes in the estimates of either of these two performance measures, we recommend that future modeling efforts account for the redistribution of recruits as a means of improving performance measure estimates.

The conservation of training companies equality constraint is

$$I_j(t) + \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} + \sum_{k=0}^{l_{\min}-1} \frac{r_j(t-k)}{x_j(t-k)} = M_j - \sum_{i=1}^t d_j(i), \quad (14)$$

where  $\sum_{k=0}^{l_{\min}-1} \frac{r_j(t-k)}{x_j(t-k)}$  and  $l_{\min} = \min \{l \in L\}$ , represents the number of "busy" training companies in week  $t$ . The summation notation counts "busy" training companies from the current week  $t$  ( $k=0$ ), backward in time, to week  $t-l+1$  ( $k=l-1$ ), where  $l$

in any week  $t$  represents the training cycle length decision  $l$  (see (10) above) from the set

$$L \in \{ (10), (10,9), (10,9,8) \} \text{ for } \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)}.$$

The training "quality" measure for one period, denoted by the instructor-to-student ratio for week  $t$  of year  $j$  and assuming one instructor per training company for simplicity, is given by

$$g[t, x_j(t)] = \frac{1}{x_j(t)}. \quad (15)$$

The following training quality performance index is used for both the objective function criterion in the optimal decision model of Chapter 3, and to compare competing feasible training resource schedules generated by the heuristic methods of Chapter 4:

$$J = \sum_{t=1}^{T_j-1} \frac{1}{x_j(t)}. \quad (16)$$

We note that the following variables and parameters are integers:

$$\left\{ I_j(t), x_j(t), y_j(t), d_j(t), D_j^*(t), D_j, r_j(t), \frac{r_j(t)}{x_j(t)} \right\}.$$

### 3. OPTIMAL DECISIONS USING DYNAMIC PROGRAMMING

In practice, *satisfactory* training resource schedules for the basic training problem presented in Chapter 2 are obtained from heuristic methods (see Section 4.1) by making scheduling decisions sequentially using "current" information. *Exact* solution methods for obtaining optimal scheduling decisions include (1) integer or mixed integer programs, (2) complete enumeration, and (3) dynamic programming. The potential combinatorial complexity that may be encountered in using these procedures to solve the basic training problem is briefly discussed below. The main reasons for choosing dynamic programming as the optimal solution method for the problem are also given.

#### 3.1 Complexity of the Basic Training Optimal Decision Problem

The complexity of using an integer (or mixed integer) program (IP) to solve the basic training problem of Chapter 2 may be illustrated by estimation of the number of basic variables required for one, of perhaps many, possible IP problem formulations. For this illustration we define the following notation: time period  $t \in (T = 96)$ , training company  $i \in (I = 130)$  and training cycle length  $j \in \{1, 2, 3\}$  that correspond to training cycle lengths of 8, 9, and 10 weeks, respectively. The basic variables for determining the strengths of the training companies (to be determined by the model) that begin a training cycle of length  $j$  in each period  $t$  are as follows:

$X_{it}$ : the number of recruits per training company  $i$  at time  $t$ , where  $X_{it}$  is integer and is bounded according to equation (1) of Chapter 2;

$$\delta_{ijt} = \begin{cases} 1, & \text{if in } t, \text{ company } i \text{ is started with a training cycle length of } j, \text{ and} \\ 0, & \text{otherwise.} \end{cases}$$

An integer programming formulation for this problem generates approximately  $5 \times 10^4$   $((130 \times 96) + (130 \times 3 \times 96))$  basic integer variables. Attempting to solve a problem of this size using integer programming methods could be a daunting task, if not a prohibitive one.

Another way to solve this problem is to completely enumerate all possible decision sequences of company strengths  $x_j(t)$  and training cycle lengths  $y_j(t)$ , and then select the optimal sequence as the one that maximizes our training "quality" objective function (see equation (16) of Chapter 2). This decision sequence for the real-world planning horizon of 96 periods is denoted by

$$\left\{ \begin{array}{l} x_1(1), x_1(2), \dots, x_2(96); \\ y_1(1), y_1(2), \dots, y_2(96) \end{array} \right\}.$$

An (upper bound) estimate of the possible number of decision sequences that may be enumerated for the real-world problem, based on 101 company strength values and 3 training cycle length values, is  $(101 \times 3)^{96}$ , or approximately  $1.65 \times 10^{238}$ . Even though many of these decision sequences are infeasible according to the problem's state feasibility constraints (see (5) of Chapter 2), complete enumeration of the problem is not possible.

A combinatorial explosion of the *state space* also occurs when attempting to obtain an optimal solution to the real-world basic training problem using dynamic programming. For example, the training cycle lengths of the basic training problem create a situation where not all the information that is needed (required) to make decisions in the "current" period can be summarized in the state variable  $I_j(t)$  that describes the evolution of the basic training system. To make the necessary time-lagged information available for decision making in the current period, the framework of the



basic training problem must be reformulated using the technique of *state augmentation* (see Section 3.3 for details). The augmented state space for a single period of the basic training problem (for 1988 training data) requires enumeration of the following state variables:

- number of idle training companies  $I_j(t+1)$ : 130;
- company strength values  $x_j(t-1), \dots, x_j(t-9)$   
for the augmented state:  $(101)^9$ ;
- training cycle lengths  $y_j(t)$ : 3.

This generates an (upper bound) estimate of the size of the state space, for each period  $t$ , of  $3 \times 130 \times (101)^9$ , or approximately  $4.27 \times 10^{20}$  (427 million trillion) possible states. Although dynamic programming substantially reduces the amount of enumeration required to obtain an optimal solution by (1) avoiding decision sequences that cannot possibly be optimal and (2) solving the problem one stage at a time, the potential size of the augmented state space for the real-world problem, or for a reduced problem (see Section 3.5 and Table 7 of Section 6.2) remains quite large. Complete enumeration of the augmented state space for the real-world problem is not possible.

The essential difference between the integer (or mixed integer) programming approach and dynamic programming is that with the integer programming method only *one* optimal decision sequence is ever generated. In dynamic programming, however, *many* optimal decision sequences (plus the objective function value for each sequence) are generated for whatever initial state and initial decision are specified. With dynamic programming it is also possible to obtain *partial* scheduling solutions for "unexpected" state conditions that may occur at any stage of the planning horizon by appealing to Bellman's *Principle of Optimality* [3, p.12]. Partial scheduling solutions are generated by

(1) "re-initializing" the problem with the unexpected state and decision for the stage where the unanticipated result occurred, and (2) using the pre-computed optimal scheduling decisions for successive stages to obtain the optimal decision sequence for the remaining periods of the planning horizon. These situations cannot, in general, be handled by integer programming in a straight forward manner. Additionally, DP is well suited to modeling the *dynamics* of the basic training system, and if stochastic variables are introduced for recruit arrivals and recruit failures (in future studies), then dynamic programming is the only method, in general, that can be used for sequential decision making. For the reasons given above, the decision was made to formulate an optimal decision model for the basic training problem using dynamic programming.

### 3.2 The Basic Training Optimal Decision Problem

The dynamic programming (DP) formulation of the basic training problem follows Bertsekas' [3] formulation of an inventory control problem. The dynamic system for the inventory control problem is a model fitting the general form of

$$x_{t+1} = f_t(x_t, u_t, w_t), \quad t = 1, 2, \dots, T-1, \quad (17)$$

where the planning horizon is divided into  $T$  time periods. Notation for the model is as follows:

- $t$ : index for  $T$  identical discrete time periods;
- $x_t$ : state variable that summarizes information needed for optimization decisions;
- $x_t$  represents the quantity of some item on-hand at the beginning of period  $t$  in the inventory control problem;

$u_t$ : decision variable denoting the decision made in period  $t$  (e.g., the amount of the item to produce or order in period  $t$ );

$w_t$ : random parameter denoting a disturbance that perturbs the system. In the inventory control problem  $w_t$  represents the level of random demand in period  $t$  for an item, and is specified by a probability distribution.

Specifically, the state of the inventory control system evolves according to the balance equation

$$x_{t+1} = x_t + u_t - w_t. \quad (18)$$

The inventory control problem is solved by determining the sequence of functions  $\mu_t(x_t)$ , for  $t = 1, 2, \dots, T-1$ , that map the current state  $x_t$  to production decisions  $u_t$  to meet random demand  $w_t$ , with the objective of minimizing a specified performance measure of system costs in each period  $t$ . The sequence of functions that map states to decisions (actions), thus constituting an inventory control policy, is given by

$$\pi = \{ \mu_1, \mu_2, \dots, \mu_{T-1} \}.$$

The random parameter  $w_t$  makes cost, in general, a random variable.  $E_\pi$  denotes the expected cost of the inventory system over the planning horizon (see Bertsekas [3], 12).

When the system costs are additive over time the total *expected* cost is summarized by

$$J_\pi(x_1) = \min_{\pi \in \Pi} E_{\pi} \left\{ g_T(x_T) + \sum_{t=1}^{T-1} g_t[x_t, \mu_t(x_t), w_t] \right\}, \quad (19)$$

where system cost incurred at each stage  $t$  of the inventory control problem includes setup cost, production cost and inventory holding cost, and is denoted by  $g_t(x_t, \mu_t(x_t), w_t)$ . The cost in the last stage  $T$ , denoted by  $g_T(x_T)$ , assumes that no production or ordering decision is made in that stage.

The set of possible inventory levels  $x_t$  at each stage belongs to an inventory space denoted by  $I_t$ , and the production decisions  $u_t$  belong to a nonempty subset of feasible production decisions  $\mathcal{U}_t(x_t)$  of the complete space of production decisions  $\mathcal{P}_t$ . The notation  $\mathcal{U}_t(x_t)$  means that the elements belonging to  $\mathcal{U}_t$  depend on the state  $x_t$  and the period  $t$ . If, for all  $t$ ,  $x_t \in I_t$  and  $\mu_t(x_t) \in \mathcal{U}_t(x_t)$ , then the inventory control policy  $\mu_t(x_t)$  for stage  $t$  is said to be *admissible*. The set of all admissible inventory control policies is denoted by  $\Pi$ .

The *optimal decision* problem is to determine the admissible inventory control policy  $\pi^*$  that minimizes the cost functional (19), for any initial state  $x_1$ . The optimal inventory control policy is denoted by

$$\pi^* = \{ \mu_1^*, \mu_2^*, \dots, \mu_{T-1}^* \},$$

and the corresponding optimal cost function is

$$J_{\pi^*}(x_1) = \min_{\pi \in \Pi} J_{\pi}(x_1).$$

In this case, the model of the inventory control problem must be formulated so that the inventory control policies,  $\mu_1, \mu_2, \dots, \mu_{T-1}$ , minimize expected costs, however, in the case of "rewards," the objective function is maximized.

Although the structure of the basic training problem is similar in some ways to the inventory control problem described above, there are significant differences between the

two problems as well. Bertsekas [3] presents a general dynamic programming formulation of the inventory control system that evolves according to (18). However, formulating a model of the basic training problem within the framework of dynamic programming requires making preliminary assumptions concerning the structure of the basic training problem. To begin, we remove the training company deactivation decision from the DP formulation of the basic training problem by requiring training company deactivation decisions  $d_j(t)$  to be made prior to implementing the DP algorithm. Other important elements of problem structure are discussed below.

### *Stages*

In the basic training problem, stages are specified by week  $t$  and year  $j$ . The planning horizon  $T_j$  consists of a finite number of identical, discrete time periods where  $t \in \{1, 2, \dots, T_j\}$  and  $j \in \{1, 2, \dots, J\}$ .

### *Problem Decisions and Scheduling Policy*

In the basic training problem, company strength  $x_j(t)$  and training cycle length  $y_j(t)$  decisions are made at the beginning of period  $t$  for  $t = 1, 2, \dots, T_j - 1$  for all training companies that begin training that period. A sequence of such decisions is represented by

$$\left\{ \begin{array}{l} x_1(1), x_1(2), \dots, x_J(T-1); \\ y_1(1), y_1(2), \dots, y_J(T-1) \end{array} \right\}.$$

The decision spaces for  $x_j(t)$  and  $y_j(t)$  are denoted by  $\Omega$  and  $\Lambda$ , respectively, where  $x_j(t) \in \Omega$  and  $y_j(t) \in \Lambda$ . The decision spaces  $\Omega$  and  $\Lambda$  consist of the bounded sets of integers specified by the company strength constraint  $\underline{x} \leq x_j(t) \leq \bar{X}$  and the training cycle constraint  $\underline{y} \leq y_j(t) \leq \bar{Y}$ . The subsets of feasible decisions to take at

each stage  $t$ ,  $x_j(t)$  and  $y_j(t)$ , are denoted by  $\mathcal{X}_j[t, I_j(t)] \subset \Omega$  and  $\mathcal{Y}_j[t, I_j(t)] \subset \Lambda$ , where feasible decision elements belonging to these two subspaces depend on both the stage  $t$  and the state  $I_j(t)$  of the basic training system; see below.

### *State of the System*

The state of the basic training system is characterized by the number of idle training companies  $I_j(t)$  that are available at the beginning of each period  $t$  to start training new recruits.

### *State Transition Equation*

The state of the basic training system evolves according to the following balance equation for idle training companies:

$$I_j(t+1) = f_j[t, I_j(t), x_j(t)] = I_j(t) + \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} - \frac{r_j(t)}{x_j(t)}, \quad (20)$$

where  $f_j[t, I_j(t), x_j(t)]$  is explicitly defined as an equivalent representation of the right hand side of (20).

The ratio  $\frac{r_j(t)}{x_j(t)}$  is an estimate of the number of companies to begin training in week  $t$  of year  $j$ , where  $r_j(t)$  gives the expected number of recruits to arrive for training each week. The sum  $\sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)}$  represents the number of companies that become available at the beginning of week  $t+1$  to start (another) training cycle having just completed one that began either eight, nine, or ten weeks earlier (see Section 2.1 for a

discussion of *compressing-the-load*). The possible values for  $l \in L$  are contained within the set  $L \in \{ (10), (10,9), (10,9,8) \}$  (see equation (10), Section 2.3, for further details).

To maintain the integer value of  $I_j(t+1)$ , both  $\frac{r_j(t)}{x_j(t)}$  and  $\sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)}$  are rounded as follows (see Section 2.3 for further details):

For  $x_j(*) < \bar{X}$ :

$$I_j(t+1) = I_j(t) + \left\lfloor \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} \right\rfloor - \left\lfloor \frac{r_j(t)}{x_j(t)} \right\rfloor - d_j(t);$$

For  $x_j(*) = \bar{X}$ :

$$I_j(t+1) = I_j(t) + \left\lceil \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} \right\rceil - \left\lceil \frac{r_j(t)}{x_j(t)} \right\rceil - d_j(t).$$

### *Demand*

In the basic training problem, demand for idle training companies is a function of the number of new recruits  $r_j(t)$  to report for training each week  $t$ . In the real-world basic training problem,  $r_j(t)$  is a random parameter. However, as explained previously in Section 2.1, recruit arrivals,  $r_j(t)$ , is estimated ahead of time for each week  $t$  and year  $j$  of the planning horizon  $T_j$ , given an annual recruiting target  $R_j$  for each year  $j$ . The absence of random disturbances in the basic training problem makes this version of the problem completely deterministic; another major distinction between the basic training problem and the inventory control problem where demand is a random parameter specified by a probability distribution.

However, from (20) we note that requirement for training companies, as represented by  $\frac{r_j(t)}{x_j(t)}$ , also depends upon  $x_j(t)$ , the decision to take in period  $t$ . This dependence is illustrated by means of two simple examples. In both of these examples, superscripts 1 and 2 denote the two cases we are comparing.

Example 1: Dependence of Training Company Demand on Recruit Arrivals.

Given:  $r_j^1(t) = 3000$ ,  $r_j^2(t) = 4000$  and  $x_j(t) = 200$ .

$$\text{Demand} \left[ r_j^1(t) \right] = \frac{r_j^1(t)}{x_j(t)} = 15;$$

$$\text{Demand} \left[ r_j^2(t) \right] = \frac{r_j^2(t)}{x_j(t)} = 20.$$

Example 2: Dependence of Training Company Demand on Company Strength Decision.

Given:  $r_j(t) = 3000$ ,  $x_j^1(t) = 150$  and  $x_j^2(t) = 250$ .

$$\text{Demand} \left[ x_j^1(t) \right] = \frac{r_j(t)}{x_j^1(t)} = 20;$$

$$\text{Demand} \left[ x_j^2(t) \right] = \frac{r_j(t)}{x_j^2(t)} = 12.$$

### *Objective Function*

In dynamic programming, the sequential decision process seeks to optimize an appropriately chosen objective function over the entire planning horizon  $T$ . For the basic



training system, the problem is to find the optimal scheduling policy  $\pi^*$  that yields an optimal objective function value  $J_{\pi^*}$  for any fixed initial condition  $I_1(1)$ , over all admissible (feasible) candidate policies  $\pi$  in  $\Pi$ , the set of all admissible scheduling policies.  $J_{\pi}$  for the basic training problem is defined in a manner similar to the general cost function given in (19) with one major difference. The absence of a random parameter in the basic training problem (see *Demand* above), makes it possible to optimally solve the problem by making the proper choice of company strength  $x_j(t)$  and training cycle length  $y_j(t)$  decisions for each period  $t$ , and for any initial condition, without computing the expected value. In this (deterministic) case, the optimal company strength and training cycle length decisions for each possible state of the basic training system can be pre-computed for each stage so that it is not necessary to "wait" until the "current" period to determine the decision to take (in that period) (see [3, p.9]).

For the deterministic basic training problem, optimizing the objective function  $J_{\pi}$  over all admissible sequences of decisions leads to the same optimal objective function value  $J_{\pi^*}$  as optimizing over all admissible system control policies  $\{ \mu_1, \mu_2, \dots, \mu_{T-1} \}$  (following the notation of the inventory control problem). For the sake of brevity, the notation given above for policies will be used for admissible decision sequences denoted by

$$\pi = \left\{ \begin{array}{l} x_1(1), x_1(2), \dots, x_J(T-1); \\ y_1(1), y_1(2), \dots, y_J(T-1) \end{array} \right\},$$

and the set of all such sequences will be also denoted by  $\Pi$ . The optimal sequence of decisions  $\pi^*$  is the one that maximizes the basic training objective function (see (21) below) for a fixed initial state  $I_1(1)$  where

$$J_{\pi^*}[I_1(1)] = \max_{\pi \in \Pi} J_{\pi}[I_1(1)].$$

For the basic training problem, the appropriateness of an objective function depends, in large measure, upon the goals of the analysis and the needs of decision makers. Two objective functions are considered here.

First, training program managers may be interested in obtaining an optimal training schedule that maximizes the quality of training. In this case, an appropriate objective function is one that incorporates a measure of training quality. As discussed previously in Section 2.1, one measure of training quality that accounts for practical aspects of scheduling training companies is the *instructor-to-student ratio*. When each training company is of equal size (see Section 2.1, *Dynamics of Varying Company Strength*), then maximizing the instructor-to-student ratio (i.e., minimizing company strengths) is equivalent to minimizing idle training companies. If we assume one instructor per training company for simplicity, then for each sequence of decisions  $\pi \in \Pi$ , a corresponding value for  $J_{\pi}$ , which provides a measure of quality to be maximized, is given by

$$J_{\pi}[I_1(1)] = \sum_{t=1}^{T_f-1} \frac{1}{x_j(t)}. \quad (21)$$

One possible objective that is an alternative to maximizing the quality of training (via the instructor-to-student ratio) is to minimize the cost of basic training. For simplicity, only four types of training costs are considered in illustrating this objective. Notation for the formulation of the cost function is

$C^V$ : variable cost per idle training company;

$C^{FS}$ : fixed "setup" cost assessed whenever companies begin a training cycle;

$C^{VC}$ : variable training cost per active training company;

$C^{VR}$ : variable training cost per recruit;

$P_j(t)$ : indicator variable, where  $P_j(t) \in \{0, 1\}$ .  $P_j(t)$  is zero when no training companies start training in period  $t$ , and one otherwise.

For this formulation, the problem is to determine the number of companies (and corresponding training company strengths) to start training each week (if any) that minimizes

$$J_\pi[I_1(0)] = C^{VI}I_J(T) + C^{FS}P_J(T) + C^{VR}r_J(T) + \sum_{t=0}^{T_J-1} \left[ C^{VI}I_j(t) + C^{FS}P_j(t) + C^{VC} \frac{r_j(t)}{x_j(t)} + C^{VR}r_j(t) \right], \quad (22)$$

subject to the system constraint  $I_j(t+1) = f_j[t, I_j(t), x_j(t)]$ , where  $t = 0, 1, \dots, T_J - 1$ ,

$f_j[t, I_j(t), x_j(t)]$  is explicitly defined by (20), and  $\frac{r_j(t)}{x_j(t)}$  is rounded according to the

rules given above (see *State Transition Equation*).

### 3.3 State Augmentation

In the inventory control problem, past actions are summarized by the state variable  $x_{t+1}$  which, together with current demand, is the necessary information needed in period  $t+1$  to make the ordering or production decision. Unfortunately, in the basic training problem, company strength  $x_j(t)$  and cycle length  $y_j(t)$  decisions depend upon past information that cannot be summarized in  $I_j(t+1)$  alone. This is due to the

dependence of decisions made in the current period on past decisions on strengths and cycle lengths of training companies; see (20).

To illustrate the interdependence of company strength decisions, assume for the moment, that cycle lengths are fixed at two periods, creating a single period lag in the dynamic model (here  $y_j(t) = 2$  and is therefore not a decision variable). Then the state of the basic training system evolves according to the following balance equation

$$I_j(t+1) = I_j(t) + \frac{r_j(t-1)}{x_j(t-1)} - \frac{r_j(t)}{x_j(t)}. \quad (23)$$

From (23), we see that  $I_j(t)$  of the simplified system does not provide sufficient information to enable the company strength decision  $x_j(t)$  to be made in period  $t$ , or to compute (23). From previous discussion (see Sections 2.1, 2.3 and 3.1),  $r_j(t)$  and  $r_j(t-1)$  are known, and the decision  $x_j(t)$  depends (in this case) upon the decision  $x_j(t-1)$  from the previous period. To illustrate the dependence of the current decision in time  $t$  on the (past) decision in  $t-1$ , consider the following example, where superscripts 1 and 2 denote the two cases that are being compared.

Given:  $r_j(t) = 2000$ ,  $r_j(t-1) = 3000$ ,  $x_j^1(t-1) = 150$  and  $x_j^2(t-1) = 250$ .

Demands for training companies for the two cases are:

$$\frac{r_j(t-1)}{x_j^1(t-1)} = 20 \quad \text{and} \quad \frac{r_j(t-1)}{x_j^2(t-1)} = 12.$$

Substituting these values into (23) yields following two cases:

$$\text{Case 1: } I_j^1(t+1) = I_j^1(t) + \frac{r_j(t-1)}{x_j^1(t-1)} - \frac{r_j(t)}{x_j^1(t)} = I_j^1(t) + 20 - \frac{2000}{x_j^1(t)};$$

$$\text{Case 2: } I_j^2(t+1) = I_j^2(t) + \frac{r_j(t-1)}{x_j^2(t-1)} - \frac{r_j(t)}{x_j^2(t)} = I_j^2(t) + 12 - \frac{2000}{x_j^2(t)}.$$

Thus the decisions in the current period,  $x_j^1(t)$  and  $x_j^2(t)$ , obviously depend upon decisions  $x_j^1(t-1)$  and  $x_j^2(t-1)$  made previously in period  $t-1$ . In other words, the structure of the illustrative problem induces a one-period time lag in the state of the system. Therefore, in general, decisions made in period  $t-1$  impact both the future state of system in period  $t$  and the allowable range of feasible decisions for period  $t$ . In order to apply dynamic programming to a problem of this type, it is necessary to reformulate the problem using a method called *state augmentation* [3]. This method makes it possible to include information from previous periods by augmenting the state space with the past information, as needed, to make the decision  $x_j(t)$  in the current period  $t$ .

Therefore, for the illustrative problem, an additional state variable  $s_j(t) = x_j(t-1)$ ,  $t = 1, 2, \dots, T_j - 1$ , is introduced to form the augmented state problem

$$\begin{bmatrix} I_j(t+1) \\ s_j(t+1) \end{bmatrix} = \begin{bmatrix} f_j[t, I_j(t), x_j(t), s_j(t)] \\ x_j(t) \end{bmatrix} = \begin{bmatrix} I_j(t) + \frac{r_j(t-1)}{s_j(t)} - \frac{r_j(t)}{x_j(t)} \\ x_j(t) \end{bmatrix}, \quad (24)$$

where  $s_j(1) = x_j(0)$  is the (last) company strength decision from the "previous" planning horizon. Defining the new (augmented) state of the system as  $\tilde{I}_j(t) = [I_j(t), s_j(t)]$ , and substituting  $\tilde{I}_j(t)$  into (24) yields the reformulated problem without a time lag:

$$\tilde{I}_j(t+1) = \tilde{f}_j[t, \tilde{I}_j(t), x_j(t)], \quad (25)$$

where  $\tilde{f}_j[t, \tilde{I}_j(t), x_j(t)]$  is defined by the right hand side of (24).

In terms of the original variables, and for a general one-stage reward function  $g_j[t, *, *]$ , the recursive DP algorithm is given by

STEP 1:

$$J_J[T, I_J(T), x_J(T-1)] = g_J[T, I_J(T), x_J(T-1)],$$

STEP 2:

$$\begin{aligned} J_J[T-1, I_J(T-1), x_J(T-2), x_J(T-1)] = \\ \max_{x_J(T-1) \in X_J[T-1, I_J(T-1), x_J(T-2)]} \left\{ g_J[T-1, I_J(T-1), x_J(T-2), x_J(T-1)] \right. \\ \left. + J_J[T, f_J(T-1, I_J(T-1), x_J(T-2), x_J(T-1)), x_J(T-1)] \right\}, \end{aligned}$$

and proceeding in this manner,

STEP  $t$ :

$$J_j[t, I_j(t), x_j(t-1), x_j(t)] = \max_{x_j(t) \in X_j[t, I_j(t), x_j(t-1)]} \left\{ g_j[t, I_j(t), x_j(t-1), x_j(t)] + J_j[t+1, f_j(t, I_j(t), x_j(t-1), x_j(t)), x_j(t)] \right\},$$

for  $t = 1, 2, \dots, T_j - 2$  and  $j = 1, \dots, J$ .

Now, we consider the more complicated case where training cycle length is fixed at  $y_j(t) = 10$  creating a nine-period time lag. Including additional variables to the problem can significantly increase both the number of computations required to generate an optimal solution, and the amount of computer memory required. Therefore, the state space is augmented by only the minimum number of variables necessary to make a decision in each period. We begin by (initially) adding only one state variable to the problem,  $s_j^1(t) = x_j(t-9)$ . The augmented system becomes

$$\begin{bmatrix} I_j(t+1) \\ s_j^1(t+1) \end{bmatrix} = \begin{bmatrix} f_j[t, I_j(t), x_j(t), s_j^1(t)] \\ x_j(t-8) \end{bmatrix} = \begin{bmatrix} I_j(t) + \frac{r_j(t-9)}{s_j^1(t)} - \frac{r_j(t)}{x_j(t)} \\ x_j(t-8) \end{bmatrix}. \quad (26)$$

However, since  $x_j(t-8)$  is not stored in either  $s_j^1(t)$  or  $I_j(t)$ , then additional information on  $x_j(t-8)$  is required by the decision model. Therefore, a second state variable  $s_j^2(t) = x_j(t-8)$  is added which further augments the system:

$$\begin{bmatrix} I_j(t+1) \\ s_j^1(t+1) \\ s_j^2(t+1) \end{bmatrix} = \begin{bmatrix} f_j[t, I_j(t), x_j(t), s_j^1(t)] \\ s_j^2(t) \\ x_j(t-7) \end{bmatrix} = \begin{bmatrix} I_j(t) + \frac{r_j(t-9)}{s_j^1(t)} - \frac{r_j(t)}{x_j(t)} \\ s_j^2(t) \\ x_j(t-7) \end{bmatrix}. \quad (27)$$

Similarly, from (27) additional information on  $x_j(t-7)$  is required. Continuing in this fashion, the minimally augmented system is given by

$$\begin{bmatrix} I_j(t+1) \\ s_j^1(t+1) \\ s_j^2(t+1) \\ s_j^3(t+1) \\ s_j^4(t+1) \\ s_j^5(t+1) \\ s_j^6(t+1) \\ s_j^7(t+1) \\ s_j^8(t+1) \\ s_j^9(t+1) \end{bmatrix} = \begin{bmatrix} I_j(t) + \frac{r_j(t-9)}{s_j^1(t)} - \frac{r_j(t)}{x_j(t)} \\ s_j^2(t) = x_j(t-8) \\ s_j^3(t) = x_j(t-7) \\ s_j^4(t) = x_j(t-6) \\ s_j^5(t) = x_j(t-5) \\ s_j^6(t) = x_j(t-4) \\ s_j^7(t) = x_j(t-3) \\ s_j^8(t) = x_j(t-2) \\ s_j^9(t) = x_j(t-1) \\ x_j(t) \end{bmatrix}. \quad (28)$$

One can think of  $\{s_j^1(t), s_j^2(t), \dots, s_j^9(t)\}$  as "registers" for temporarily storing the required information as the system evolves.

Denoting  $\tilde{I}_j(t) = [I_j(t), s_j^1(t), s_j^2(t), \dots, s_j^9(t)]$ , then the reformulated system is  $\tilde{I}_j(t+1) = \tilde{f}_j[t, \tilde{I}_j(t), x_j(t)]$ , where  $\tilde{f}_j[t, *, *]$  represents the right hand side of (28).



The DP algorithm for the nine-period time lag problem is given below in Section 3.4 (see *DP Algorithm*).

Multiple training cycle lengths may be incorporated into the model by augmenting the state of the system with additional variables denoting the values of  $y_j(t-10)$ ,  $y_j(t-9)$  and  $y_j(t-8)$ . Using the state augmentation technique to account for past training cycle length decisions follows the same state augmentation steps explained above for company strengths.

### 3.4 DP Model of the Basic Training Problem

We summarize here the DP model for the problem with a nine-period time lag, where the training cycle length decision  $y_j(t)$  is removed from the decision model by fixing the training cycle lengths at the upper bound  $y_j(t) = 10$ . The problem is to optimize an objective function, denoted by  $J_\pi^*$ , by determining the optimal company strength policy  $\pi^* = \{x_1^*(1), x_1^*(2), \dots, x_j^*(T-1)\}$ . The problem formulation and the DP algorithm given below are expressed in terms of the new state  $\tilde{I}_j(t)$ .

*Objective Function:*

$$\begin{aligned} &\text{maximize } J_\pi[1, \tilde{I}_1(1)] \\ &\pi \in \Pi \end{aligned}$$

*Subject to:*

a) Decision Variable Constraints:

$$\underline{x} \leq x_j(t) \leq \bar{X}: \quad \text{training company strength;}$$

b) State Variable Constraints:

$$\tilde{I}_j(t+1) = \tilde{f}_j[t, \tilde{I}_j(t), x_j(t)]: \quad \text{state transition;}$$

$$I_j(t) \leq \bar{I} \quad \forall (t, j): \quad \text{constraint on the number of idle companies;}$$

$$I_j(t) \geq 0 \quad \forall (t, j): \quad \text{problem feasibility constraint;}$$

$$\left\{ \tilde{I}_j(t), x_j(t), r_j(t), \frac{r_j(t)}{x_j(t)} \right\}: \quad \text{integer.}$$

*DP Algorithm:*

STEP 1:

$$J_J[T, \tilde{I}_J(T)] = g_J[T, \tilde{I}_J(T)],$$

STEP 2:

$$J_J[T-1, \tilde{I}_J(T-1)] =$$

$$\max_{x_J(T-1) \in X_J[T-1, \tilde{I}_J(T-1)]} \left\{ g_J[T-1, \tilde{I}_J(T-1), x_J(T-1)] \right.$$

$$\left. + J_J[T, \tilde{f}_J(T-1, \tilde{I}_J(T-1), x_J(T-1))] \right\},$$

STEP  $t$ :

$$J_j[t, \tilde{I}_j(t)] = \max_{x_j(t) \in X_j[t, \tilde{I}_j(t)]} \left\{ g_j[t, \tilde{I}_j(t), x_j(t)] + J_j[t+1, \tilde{f}_j(t, \tilde{I}_j(t), x_j(t))] \right\},$$

*for  $t = 1, 2, \dots, T_j - 2$  and  $j = 1, \dots, J$ .*

### 3.5 Simplifying the Augmented State Space of the Problem

An unfortunate consequence of state augmentation is the exponential explosion of the state space. Practical limitations of computing technology constrain the size of the problem that can be solved via dynamic programming. As discussed in Section 3.1, implementing the DP algorithm for the augmented basic training problem (for 1988 training data) may (potentially) require enumeration of  $3 \times 130 \times (101)^9$ , or approximately  $4.27 \times 10^{20}$  possible states, which is not possible.

However, simplifications to the problem structure may lead to a reduction in the size of state space enabling optimal scheduling decisions from DP to be compared with heuristic solutions to the simplified problem. For example, the number of admissible company strength values may be reduced by restricting intermediate values between the lower and upper bounds to be multiples of a predetermined step size. Using a company strength step of five, the number of company strength values to enumerate in each stage is reduced from 101 to 21. Furthermore, tests of realistic recruiting targets using heuristic scheduling methods (see Chapter 4) reveal that there are never more than fifty idle training companies in any period for the set of training companies tested and training

cycle lengths of ten weeks, resulting in the constraint  $I_j(t) \leq 50$ . These two assumptions significantly reduce the size of the state space for each period to  $3 \times 50 \times (21)^9$ , or approximately  $1.19 \times 10^{14}$  (119 trillion) possible states. This is still a very large problem, but six orders of magnitude smaller than the original problem. Results comparing the dynamic programming solutions to heuristic solutions for a (further) simplified problem are given in Section 6.2.

#### 4. HEURISTIC APPROACHES

An alternative approach to the dynamic programming formulation of the problem is a heuristic solution procedure. The heuristic approach presented here consists of two different heuristic procedures applied sequentially in three phases. The heuristics have been implemented in a fully operational computer-based decision support system (DSS) (see Chapter 5) for generating "good" training resource schedules in reasonable time (see *Results*, Chapter 6). Both heuristics feature an automated policy improvement algorithm motivated by the policy improvement step of dynamic programming (see Bertsekas) that is tailored to fit the unique structure of the basic training problem. Automated policy improvement eliminates the need for an analyst to interactively schedule resources; a notable improvement over existing computer-supported manual methods where scheduling is done by trial-and-error.

The two heuristics of the solution procedure have been implemented in a spreadsheet called *LOTUS 1-2-3 for Windows, Release 4*; the same software environment used previously for implementing the heuristics-used-in-practice (HUIP) (see *Preliminary Work*, Section 5.1). In addition, advanced macros provide programming flexibility for fully implementing the heuristic procedures.

##### 4.1 Overview of the Heuristic Approaches

Phase I starts with an initial training requirement for each week  $t$ , denoted by  $\{r_1(1), r_1(2), \dots, r_1(T)\}$ , that is estimated from the initial recruiting target  $R_j$  for each year  $j$ . An efficient *single-pass heuristic* (SPH) makes one forward pass through the planning horizon applying a policy iteration algorithm a finite number of times in each period  $t$  until an *initial feasible training resource schedule* is obtained (if one exists) for the currently available resources. The training resource scheduling policy in Phase I is

the sequence of decisions on company strength  $x_j(t)$  and training cycle length  $y_j(t)$  for each period  $t$ . The policy is specified by

$$\pi^1 = \left\{ \begin{array}{l} x_1^1(1), x_1^1(2), \dots, x_J^1(T-1); \\ y_1^1(1), y_1^1(2), \dots, y_J^1(T-1) \end{array} \right\},$$

where superscript 1 denotes Phase I.

Phase II considers options for changing the level of resources (e.g., deactivating training companies) available to train recruits, and is motivated by recent decisions to downsize the training installation complex. Incorporating this aspect of training base management into the heuristic approach enables different types of problems to be studied, such as evaluating how decisions that change the training base structure impact training resource scheduling. Options for deactivating training companies, specified by  $\{d_1(1), d_1(2), \dots, d_J(T)\}$ , include cutting training companies either at the beginning or the end of a year, or distributing the cuts across the planning horizon. Our example is restricted to one type of training resource (i.e., training companies), and to decisions that reduce the level of available resources. However, the model is easily modified to also consider resource level increases and multiple reusable resources.

Phase II adds another decision variable (the number of training companies to be deactivated  $d_j(t)$  in each period  $t$ ), and another state variable (the number of training companies remaining to be deactivated  $D_j^*$  for year  $j$ ) to the problem. Once model parameters are adjusted to reflect actual or potential changes to the training base, the single-pass heuristic is used (again) to return a feasible resource schedule (if one exists) for the currently available resources. If no resource changes are needed, then Phase II may be omitted. The resource scheduling policy for Phase II is

$$\pi^2 = \left\{ \begin{array}{l} x_1^2(1), x_1^2(2), \dots, x_J^2(T-1); \\ y_1^2(1), y_1^2(2), \dots, y_J^2(T-1); \\ d_1^2(1), d_1^2(2), \dots, d_J^2(T-1) \end{array} \right\},$$

where  $d_t^2(t)$  is the company deactivation decision in period  $t$ , and the superscript 2 denotes Phase II.

As explained earlier, the single-pass heuristic (SPH) used in Phase I and Phase II makes one sequential forward pass through the planning horizon to correct training company shortfalls in each period, and then stops. Experiments have shown that it is possible to improve resource schedules obtained via SPH by making additional passes through the planning horizon using a modified policy improvement step to further decrease training company strengths. This observation led to the development and implementation of a *multi-pass heuristic* (see Section 4.3). The multi-pass heuristic (MPH) that improves the resource scheduling policies obtained from the single-pass heuristic.

Phase III uses the initial feasible schedule from Phase II (or from Phase I if Phase II is omitted) as its starting point. The initial company strength scheduling policy is iteratively revised, period-by-period, using MPH that works sequentially backward through the planning horizon until no further improvements to the objective function are possible with the MPH. The final resource scheduling policy, obtained at the completion of Phase III, is given by

$$\bar{\pi}^3 = \left\{ \begin{array}{l} \bar{x}_1^3(1), \bar{x}_1^3(2), \dots, \bar{x}_j^3(T-1); \\ y_1^2(1), y_1^2(2), \dots, y_j^2(T-1); \\ d_1^2(1), d_1^2(2), \dots, d_j^2(T-1) \end{array} \right\},$$

where  $\bar{x}_j^3(t)$  denotes the "best" company strength decision obtainable in period  $t$  using the backward MPH recursion of Phase III, and  $y_j^2(t)$  and  $d_j^2(t)$  are the training cycle and training company deactivation decisions from Phase II.

The heuristic procedures presented here are based on the mathematical formulation of the basic training problem presented in Chapter 2, and assume a 96-week (two-year) planning horizon. Company strengths  $x_j(t)$  are initialized at the lower bound of 150 recruits. This establishes a *utopian* bound from above for the "training quality"

performance measure (i.e., the instructor-to-student ratio),  $\sum_{t=1}^{T_j-1} \frac{1}{x_j(t)}$ , of 0.64 (which is unachievable, in general).

The logical flows of the single- and multi-pass heuristics are diagrammed below in Figures 5-9. After the flow diagrams, a step-by-step explanation of the procedures of the two heuristics is also given.

#### 4.2 Single-Pass Heuristic (SPH)

Before Phase I begins, the training base scenario is tested to determine whether a feasible schedule exists for the scenario (see Figure 5). This is done by initializing training company strengths at their upper bound and training cycle lengths at their lower bound. If the initial scheduling shows that the feasibility constraint



$(I_j(t) \geq 0 \quad \forall (t,j))$  holds for each period of the planning horizon, then a feasible schedule exists.

Phase I seeks an initial feasible schedule (if one exists) using two decision variables (company strength  $x_j(t)$  and training cycle length  $y_j(t)$ ), and one state variable (the number of training companies idle at the beginning of each week  $I_j(t)$ ). The initial conditions for Phase I assume:

1. an annual recruiting target  $R_j$  for each year  $j$ ;
2. company strengths  $x_j(t)$  initialized at the lower bound of 150 recruits per company;
3. training cycle lengths  $y_j(t)$  initialized at the upper bound of ten weeks;
4. no changes to the number of available training companies to train recruits are considered ( $d_j(t) = 0 \quad \forall (t,j)$ ); and
5. information needed to compute  $I_1(1), I_1(2), \dots, I_1(9)$  is given in the form of

$$\sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)}, \text{ for each case.}$$

Figure 5 shows the logical flow of the Resource Scheduling Algorithm of Phase I. Figure 6 diagrams the single-pass heuristic Company Strength Policy Improvement Step, and Figure 7 outlines the steps for the training cycle adjustment procedure followed when attempting to correct a training company shortfall that could not be corrected via company strength policy improvement.

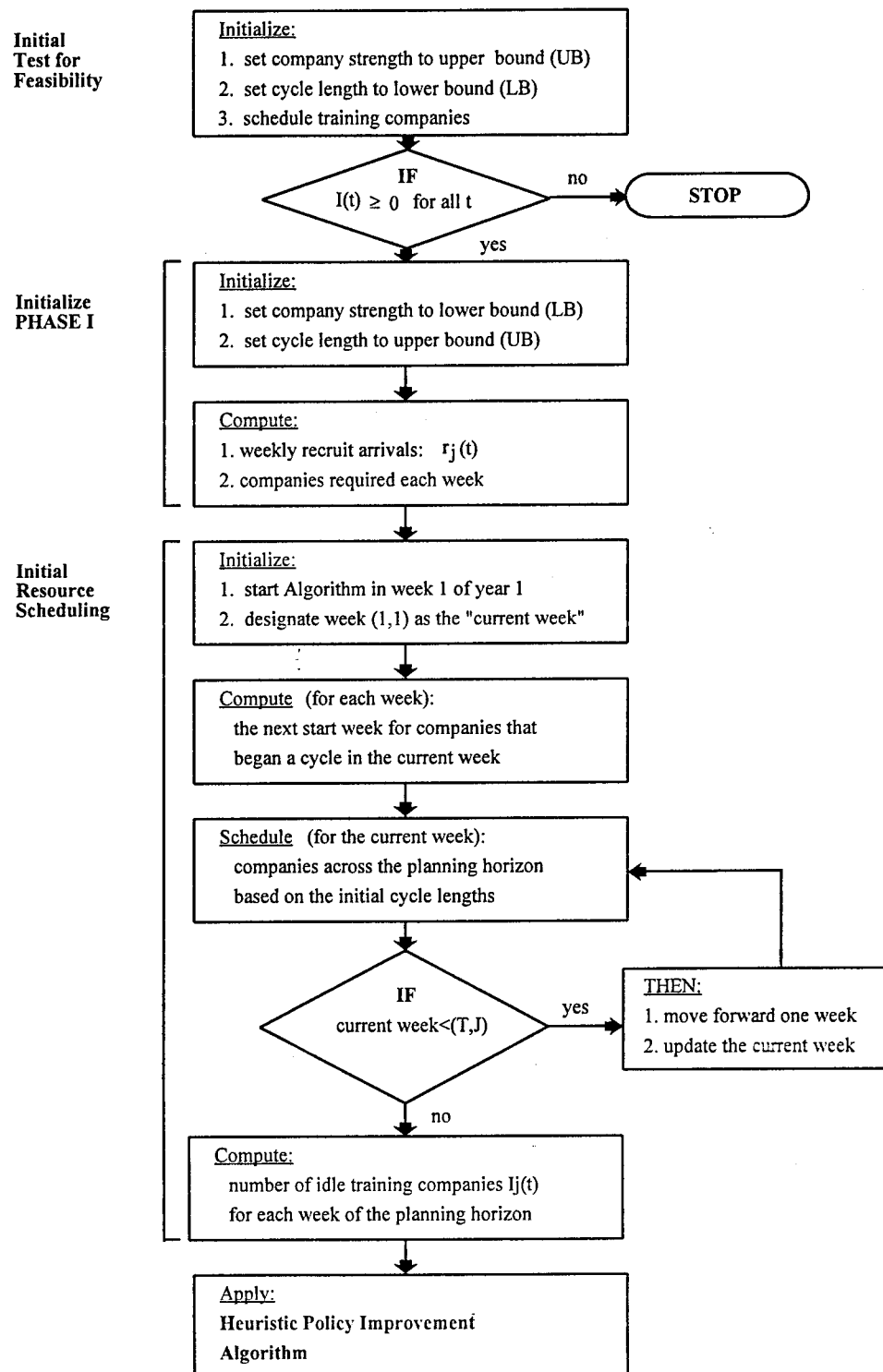


Figure 5. Phase I Initial Resource Scheduling Algorithm

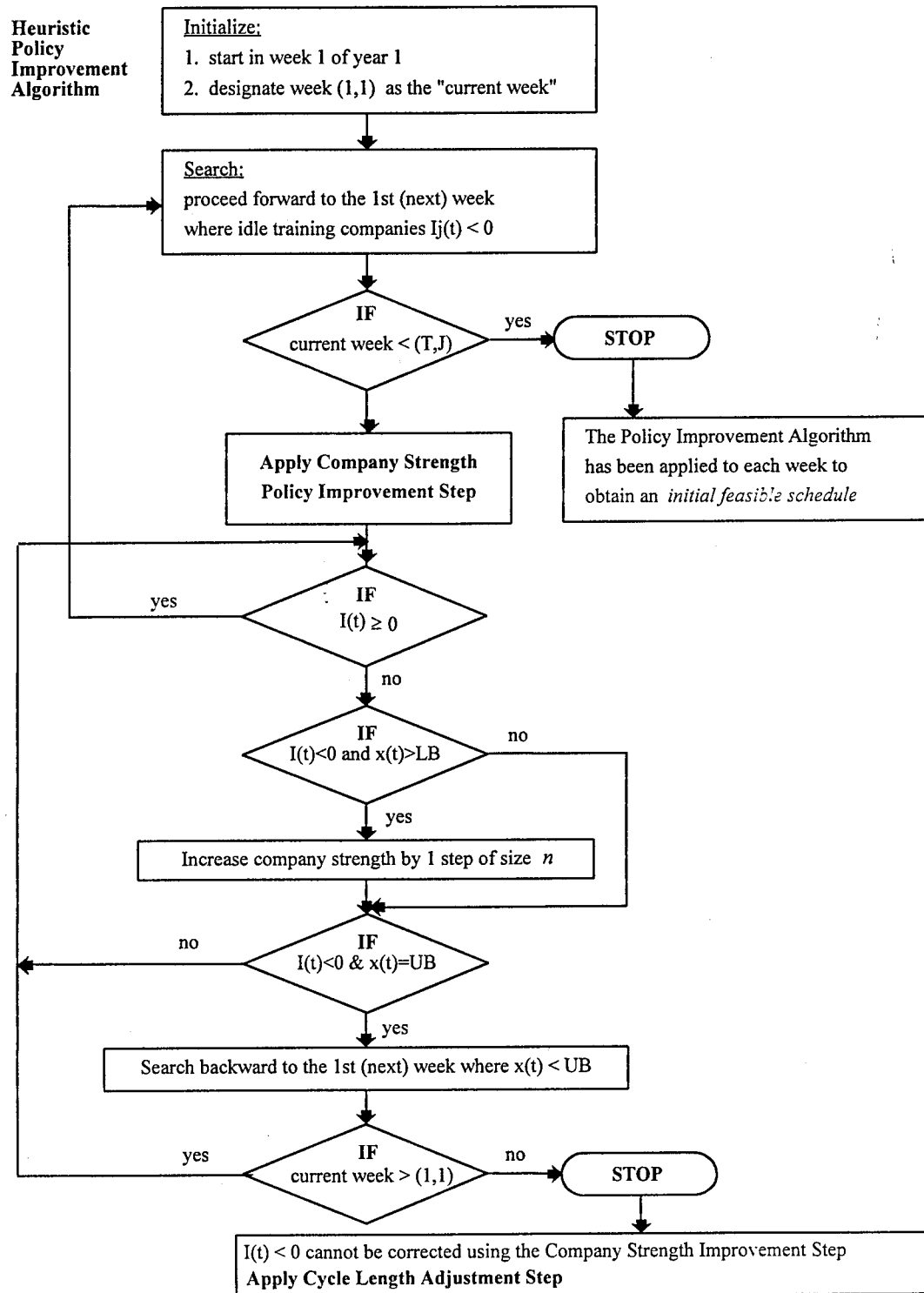


Figure 6. Phase I Single-Pass Heuristic Policy Improvement Algorithm

**Heuristic  
Policy  
Improvement  
Algorithm**  
(continued)

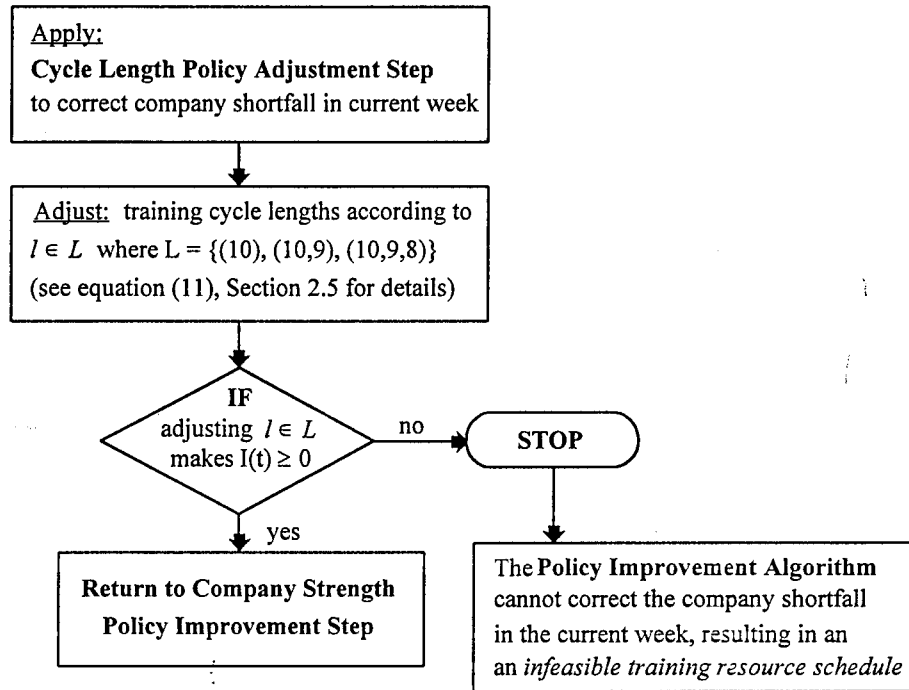


Figure 7. Phase I Training Cycle Length Policy Adjustment

*PHASE I: Finding an Initial Feasible Training Resource Schedule*

1. Initialize Phase I: set  $x_j(t) = \underline{x}$ ,  $y_j(t) = \bar{Y}$ , and  $d_j(t) = 0 \quad \forall (t,j)$ .
2. Compute weekly recruit arrivals  $r_j(t) = \delta(t) p(t) R_j \quad \forall (t,j)$  given  $R_j$ .
3. Compute companies required in each week  $\frac{r_j(t)}{x_j(t)} \quad \forall (t,j)$  as follows

$$\text{For } x_j(t) < \bar{X}: \left\lceil \frac{r_j(t)}{x_j(t)} \right\rceil;$$

$$\text{For } x_j(t) = \bar{X}: \left\lceil \frac{r_j(t)}{x_j(t)} \right\rceil.$$

**4. RESOURCE SCHEDULING ALGORITHM:**

- A. Start the RESOURCE SCHEDULING ALGORITHM in week  $1$  of year  $1$ .
- B. Designate this week as the current week  $\hat{t}$ .
- C. Get the cycle length  $y_j(\hat{t})$  for the current week  $\hat{t}$ .
- D. Compute the next start week for the companies that begin a training cycle in week  $\hat{t}$  as of week  $\hat{t} + y_j(\hat{t})$ .

E. Get  $\frac{r_j(\hat{t})}{x_j(\hat{t})}$  for week  $\hat{t}$ .

F. Schedule companies  $\frac{r_j(\hat{t})}{x_j(\hat{t})}$  to be available to start again in week  $\hat{t} + y_j(\hat{t})$ .

**IF** week  $\hat{t} + y_j(\hat{t}) \leq T_j$ ;

**THEN** go to week  $\hat{t}$ , move forward one period in the planning horizon from week  $\hat{t}$  of year  $j$  to week  $\hat{t} + 1$ , and return to STEP 4B. Otherwise

**CONTINUE.**

**IF** week  $\hat{t} + y_j(\hat{t}) > T_j$ ;

**THEN** the RESOURCE SCHEDULING ALGORITHM has completed scheduling for the entire planning horizon.

**CONTINUE.**

5. Compute:  $I_j(t+1) = I_j(t) + \sum_{l \in L} \frac{r_j(t-l)}{x_j(t-l)} - \frac{r_j(t)}{x_j(t)} - d_j(t) \quad \forall (t,j).$

6. **HEURISTIC POLICY IMPROVEMENT ALGORITHM via AUTOMATED POLICY ITERATION:**

- A. Start the POLICY IMPROVEMENT ALGORITHM in week  $l$  of year  $l$ .
- B. Proceed sequentially forward through each week of the planning horizon to the first (or next) week where there is a training company shortfall (e.g., *shortfall week: SFW*), such that,  $I_j(t+1) < 0$ .
- C. Designate this week as the shortfall week  $\hat{t}_{SFW}$ .

IF week  $\hat{t}_{SFW} + 1 > T_j$ ;

STOP; the POLICY IMPROVEMENT ALGORITHM has been applied to each week of the planning horizon resulting in an *initial feasible schedule*  $\pi^1$ . Proceed to Phase II (or to Phase III if Phase II is omitted). Otherwise

CONTINUE.

D. **COMPANY STRENGTH POLICY IMPROVEMENT STEP:**

IF  $I_j(\hat{t}_{SFW} + 1) = 0$ ;

THEN return to STEP 6B. Otherwise

CONTINUE.

IF  $I_j(\hat{t}_{SFW} + 1) < 0$  **and**  $x_j(\hat{t}_{SFW}) < \bar{X}$ ;

THEN increase company strength by one step of size  $n$ , replace

$x_j(\hat{t}_{SFW})$  by  $x_j(\hat{t}_{SFW}) + n$ , and return to STEP 6D. Otherwise

CONTINUE.

IF  $I_j(\hat{t}_{SFW} + 1) < 0$  **and**  $x_j(\hat{t}_{SFW}) = \bar{X}$  **and** week  $\hat{t}_{SFW} - i \geq 1$ , where  
 $i \in \{1, \dots, \hat{t}_{SFW} - 1\}$ ;

THEN proceed sequentially backward from week  $\hat{t}_{SFW} + 1$  to the  
 first (or next) week  $\hat{t}_{SFW} - i$ , where  $i \in \{1, \dots, \hat{t}_{SFW} - 1\}$  and  
 where  $x_j(\hat{t}_{SFW} - i) < \bar{X}$ , such that  $i$  takes on values  
 sequentially beginning with  $i = 1$ . Otherwise

**CONTINUE.**

IF the current week  $\hat{t}_{SFW} - i < 1$ ;

THEN further adjustments to company strengths cannot be made,  
 and it is not possible to correct the training company shortfall  
 in week  $\hat{t}_{SFW} + 1$  via the COMPANY STRENGTH POLICY  
 IMPROVEMENT STEP.

**CONTINUE.**

#### E. CYCLE LENGTH POLICY ADJUSTMENT STEP:

Attempt to correct the training company shortfall in week  $\hat{t}_{SFW} + 1$  by

adjusting training cycle lengths according to  $l \in L$  in  $\sum_{l \in L} \frac{r_j(\hat{t}_{SFW} - l)}{x_j(\hat{t}_{SFW} - l)}$

where  $L \in \{(10), (10,9), (10,9,8)\}$  (see equation (10), Section 2.4, for  
 details).

IF adjusting  $l \in L$  makes  $I_j(\hat{t}_{SFW} + 1) \geq 0$ ;

THEN return to STEP 6B. Otherwise

CONTINUE.

IF  $I_j(\hat{t}_{SFW} + 1) < 0$  and  $x_j(\hat{t}_{SFW}) = \bar{X} \quad \forall t \in \{1, \dots, \hat{t}_{SFW}\}$  and  
 $l = L_{MAX}$  for week  $\hat{t}_{SFW} + 1$ ;

THEN further adjustments cannot be made to either company strengths or training cycle lengths, and it is not possible to correct the training company shortfall in week  $\hat{t}_{SFW} + 1$ .

STOP; the resource scheduling policy for the current training scenario is *infeasible*.

Phase II assumes the following information from Phase I:

1. recruiting target  $R_j$ ;
2. training scenario  $\{r_1(1), r_1(2), \dots, r_j(T)\}$ ;
3. initial scheduling policy  $\pi^1$ .

The Resource Scheduling Algorithm and the Policy Improvement Algorithm (Steps 4 and 6, respectively) from Phase I are applied (again) in Phase II to find a feasible training resource schedule  $\pi^2$ . Figure 8 diagrams the single-pass heuristic for Phase II.



**PHASE II:  
Heuristic  
Policy  
Improvement  
Algorithm**

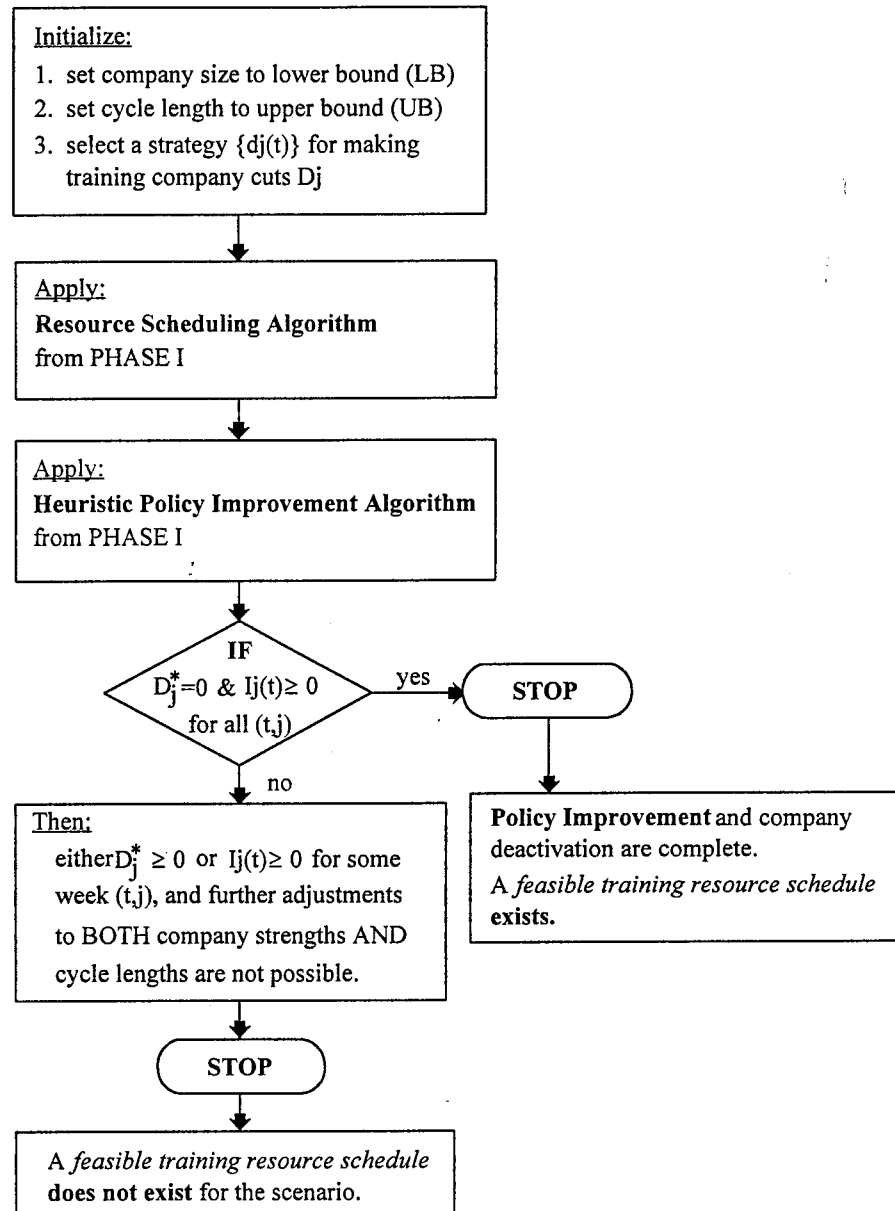


Figure 8. Phase II Single-Pass Heuristic Policy Improvement Algorithm

*PHASE II: Finding a Feasible Training Schedule Subject To Changes in Training Resource Availability.*

1. Given  $D_j$ , apply a strategy  $\{d_1(1), d_1(2), \dots, d_j(T)\}$  for reducing training companies.
2. Apply the RESOURCE SCHEDULING ALGORITHM from PHASE I.
3. Apply the HEURISTIC POLICY IMPROVEMENT ALGORITHM from PHASE I.
4. **PHASE II STOPPING RULES:**

**IF**  $D_j^*(t) = 0$  **and**  $I_j(t) \geq 0 \quad \forall (t, j);$

**STOP**; company deactivation is complete and a *feasible* resource scheduling policy  $\pi^2$  exists for the current training scenario.

Otherwise

**CONTINUE.**

**IF**  $D_j^*(t) \geq 0$  **and**  $\exists$  week  $\hat{t}_{SFW} \ni I_j(\hat{t}_{SFW} + 1) < 0$  **and** further adjustments cannot be made to either company strengths (e.g.,  $x_j(t) = \bar{X}$  for all  $t \in \{1, \dots, \hat{t}_{SFW}\}$ ) or to cycle lengths (e.g.,  $l = L_{MAX}$  for week  $\hat{t}_{SFW} + 1$ );

**THEN** it is not possible to correct the training company shortfall in at least one week, namely  $\hat{t}_{SFW} + 1$ .

**STOP**; the resource scheduling policy for the current training scenario is *infeasible*.

### 4.3 Multi-Pass Heuristic (MPH)

The starting point for Phase III assumes a feasible scheduling policy from either Phase I or Phase II, or from some other heuristic scheduling method. The multi-pass heuristic starts in week  $T_j$  of the planning horizon and proceeds backward in time to each week  $\hat{t}$  where  $I_j(\hat{t}) > 0$  and  $x_j(\hat{t}) > \underline{x}$ . In each week  $\hat{t}$ , the company strength  $x_j(\hat{t})$  is decreased by one step of size  $n$  at a time until  $I_j(\hat{t}) < 0$ . After each company strength decrement, the MPH checks each week of the planning horizon  $\{\hat{t}, \dots, T_j\}$  to ensure the last iteration of the policy improvement algorithm did not cause the current policy to become infeasible (e.g.,  $I_j(\tilde{t}) < 0$  for some  $\tilde{t} \in \{\hat{t}, \dots, T_j\}$ ). The first time that decreasing  $x_j(\hat{t})$  causes  $I_j(\tilde{t}) < 0$  in some week  $\tilde{t} \in \{\hat{t}, \dots, T_j\}$ , then the last decrement is undone by increasing company strength in week  $\hat{t}$  by one step of size  $n$  to make  $I_j(\tilde{t}) \geq 0$ . This process is repeated, step-by-step and period-by-period, until no further improvements can be made to the company strength policy, resulting in the "best" scheduling policy  $\bar{\pi}^3$  obtainable via the MPH in Phase III. Figure 9 outlines the steps of the Multi-Pass Heuristic Policy Improvement Algorithm for Phase III, that is (again) followed by the step-by-step procedures for the MPH.

**PHASE III:  
Multi-pass Heuristic  
Policy  
Improvement  
Algorithm**

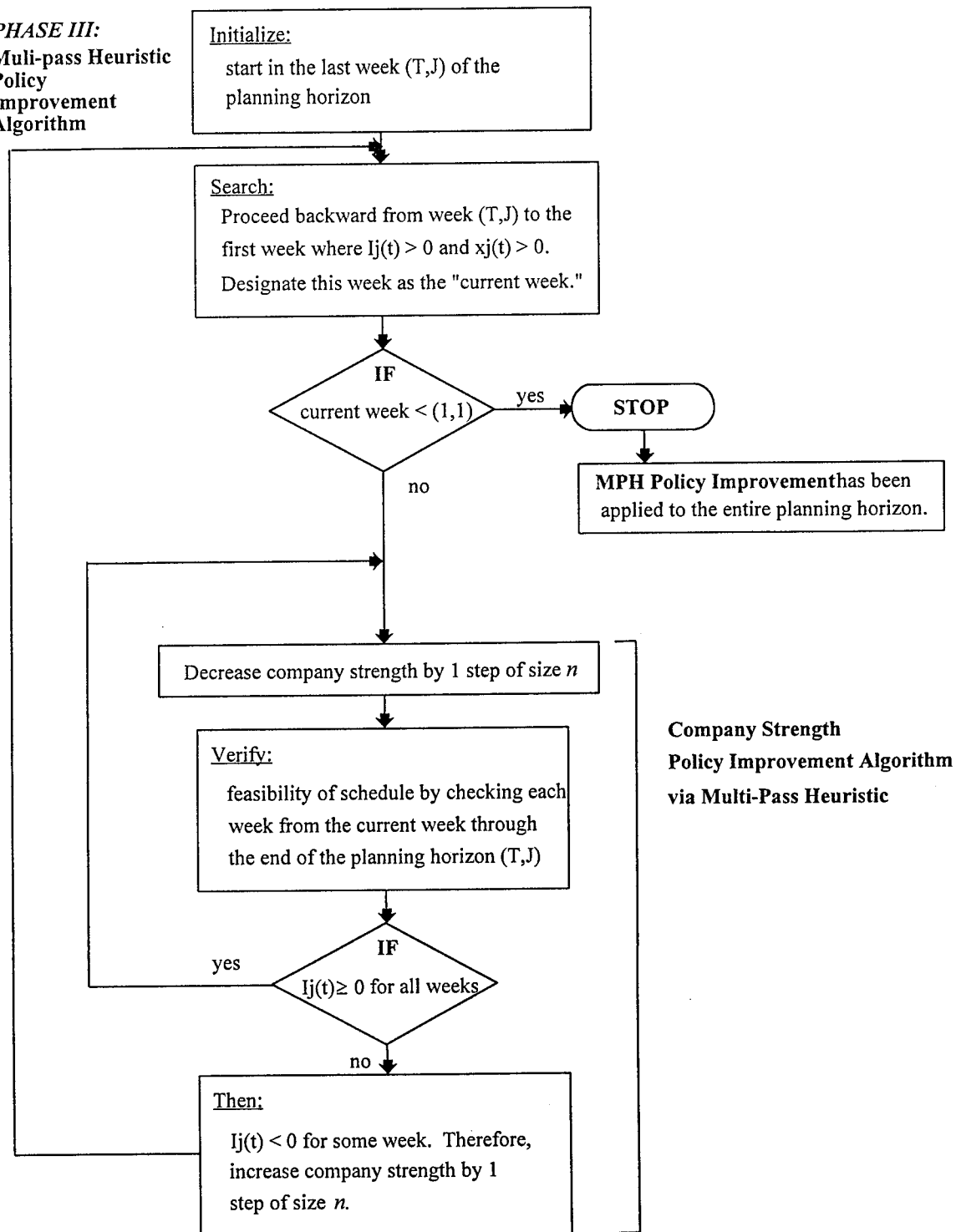


Figure 9. Phase III Multi-pass Heuristic Policy Improvement Algorithm

*PHASE III: Resource Schedule Policy Improvement via Multi-Pass Heuristic*

1. Start in week  $T$  of year  $J$ .
2. Proceed sequentially backward through the planning horizon from week  $T$  to the first (or next) week  $\hat{t}$ , where  $I_j(\hat{t}) > 0$  and  $x_j(\hat{t}) > \underline{x}$ .

IF the current week  $(\hat{t}, j) < (1, 1)$ ;

THEN the MPH COMPANY STRENGTH POLICY

IMPROVEMENT ALGORITHM has been applied to the entire planning horizon.

STOP; the policy  $\bar{\pi}^3$  is the best one obtainable using the MPH.

Otherwise

CONTINUE.

3. **COMPANY STRENGTH POLICY IMPROVEMENT ALGORITHM via MULTI-PASS HEURISTIC**

- A. Decrease company strength by one step of size  $n$  in week  $\hat{t}$  from  $x_j(\hat{t})$  to  $x_j(\hat{t}) - n$ .
- B. Verify schedule feasibility by checking each week  $\tilde{t} \in \{\hat{t}, \dots, T_j\}$ , in sequence, proceeding forward through the planning horizon from week  $\hat{t}$  to the end of the planning horizon at week  $T_j$ .

IF  $I_j(\tilde{t}) \geq 0 \quad \forall \tilde{t} \in \{\hat{t}, \dots, T_j\}$ ;

THEN return to week  $\hat{t}$  in the model and return to STEP 1 of Phase III (MPH). Otherwise

CONTINUE.

**IF**  $\exists \tilde{t} \in \{ \hat{t}, \dots, T_j \} \ni I_j(t) < 0;$

**THEN** return to week  $\hat{t}$  in the model, increment  $x_j(\hat{t})$  by one step of size  $n$  to make  $I_j(\tilde{t}) \geq 0$  , and

**RETURN** to STEP 1.

## 5. DECISION SUPPORT SYSTEM (DSS)

The logistical complexities of the Army's initial entry training program create numerous practical decision making problems. Many of these decision situations involve problems where the best solutions are not obvious. This may be due to multiple competing objectives, or situations where outcomes depend upon a sequential decision process complicated by precedence constraints, or the need to evaluate the impact of decisions over varying planning horizons.

The decision support system presented here is a robust paradigm for carefully modeling, studying, and methodically solving problems related to the Army's Basic Combat Training (BCT) program. For this reason, the decision support system is being referred to as the *Decision Support System for Army Basic Combat Training Resource Management per Year*<sup>1</sup>, or *ARMY*. Despite the current Army BCT orientation of the system, it is believed that the DSS can be extended to other Army training programs, and to training programs of other branches of military service (Navy, Air Force, Marines) as well. The *ARMY* system has been designed to support decision making at three major levels of basic training management: strategic planning at the Department of the Army (DA) level, training installation management at the U.S. Army Training and Doctrine Command (TRADOC) Headquarters, and operational control of the training program at the training installation level.

At the Army's highest level, Department of the Army, strategic planners project future recruiting objectives to ensure that the (future) force is properly manned. The determination of recruiting objectives leads to recruiting targets, which in turn drive requirements for training resources. Issues relating to military force structure complicate the strategic planning process. For example, the continued infusion of new technologies

---

<sup>1</sup>Courtesy of Professor Emmanuel Fernández-Gaucherand, dissertation coadvisor, of the Department of Systems and Industrial Engineering at the University of Arizona.

into Army systems may lead to the design of new training programs, or to substantive changes to curriculum in order to meet future needs for soldiers with more highly developed technical skills. In either case, potential changes to course lengths and class sizes of the training program may directly impact recruitment, training program throughput, training resource requirements, and training program costs. Downsizing the military may also cause significant changes to the structure of the training installation complex, such as, the consolidation of Army training programs, or the creation of "joint" training centers for training recruits from all branches of service at the same installation.

At TRADOC Headquarters, training program managers acquire and distribute training resources needed by training installations to accomplish DA training missions. Management issues include (1) determination of training loads for each training installation, (2) justification of training resource requirements and costs to DA, and (3) conducting studies and preparing plans for dealing with special training contingencies (e.g., mobilization) or potential changes to training programs (e.g., base closures, downsizing, etc.).

At the operational level, training program managers resolve resource scheduling problems that impact the efficiency and effectiveness of training program execution. Training execution problems confronting training program managers at the operational (installation) level include (1) developing precise training resource schedules that meet training requirements, (2) identifying and resolving training resource shortfalls, and (3) determination of efficient resource utilization.

The *ARMY* system currently supports analysis of the strategic, mid-level, and operational issues described above. For example, studying varying course lengths using *ARMY* is easily done since course length is a model parameter that may be specified in *ARMY* by "clicking" on the cycle length "button" of the user input screen and entering the



course length (value) from the keyboard. The *ARMY* system also features the capability for estimating cost alternatives (see Section 5.4) based on solutions generated via the single- and multi-pass heuristics of Chapter 4.

With the *ARMY* system, it is possible to easily model both *training base* changes (as indicated by varying levels of basic training companies) and *force structure* changes (as indicated by varying recruiting objectives) since available training companies and recruiting objectives are model parameters that can be easily changed by the user. Appendix B illustrates how the *ARMY* system might be used to support analysis of potential changes to the training base structure.

The DSS offers several performance measures for evaluating training base and force structure scenarios that support the decision making process, such as (1) training program quality (via the instructor-to-student ratio), (2) training program costs, (3) training program throughput, and (4) the level (quantity) of resources required to satisfy training objectives.

Finally, *ARMY* provides training program managers at training installations with a fully automated computer-based DSS for generating "good" weekly schedules of reusable training resources in reasonable time. The have (potentially) high practical value as a preliminary step in developing executable training resource schedules for day-to-day operations.

## 5.1 Preliminary Work

The *ARMY* system is based, in part, on past work by McGinnis [10] for the U.S. Army Training and Doctrine Command Headquarters. In 1987, Congress established the Base Realignment and Closure Commission (BRAC) to examine the military installation complex, determine the feasibility of consolidating or realigning military missions, report

findings, and make recommendations to Congress for partial, or complete, closure of military bases. In response to the Commission, TRADOC Headquarters initiated a project to develop computer-based methods for studying alternatives (called *training base scenarios*) to re-structure and re-engineer the training installation complex, and to assess the likely impact of such changes on TRADOC's ability to meet future training missions, and to maintain training readiness. The project, completed in 1989, consisted of a computer-supported, three-level modeling framework, henceforth referred to as the *1989 DSS*, for studying training base scenarios.

The three levels of the modeling framework, *long-term aggregate*, *mid-term aggregate*, and *near-term disaggregate*, represent distinct top-down views of the initial entry training process. The planning horizons are four, two, and one year(s), respectively. The *aggregate* view of initial entry training models training companies from all installations as a single group (see Figure 2), whereas, the *disaggregate* view models each training installation and training program separately (see Figure 3).

Long-term aggregate analysis of training base scenarios begins by estimating the annual recruit throughput per training company for each phase of initial entry training (e.g., the total number of recruits to be trained by an initial entry training company in a given year). Next, the number of training companies required to train new recruits each year (based on forecasted annual recruiting targets) is estimated as the annual recruiting target divided by the annual company throughput. Then, the number of training companies available to train recruits is estimated using "best-case" assumptions (e.g., training company cuts are always made at year's end, and company strengths for surge and nonsurge periods are 250 and 200 recruits per company, respectively). Comparing the estimated number of *required* versus *available* training companies determines the feasibility of a training scenario (for the assumptions used). If the number of companies

*required* is greater than the number of companies *available in any year* of the four-year planning horizon, then the training base scenario is *infeasible*. Although not a sophisticated method, long-term aggregate analysis proved to be very useful for quickly and safely eliminating infeasible training base scenarios from further consideration.

In mid-term aggregate analysis, separate models of each training phase (BCT, AIT, OSUT) are used to generate a weekly training schedule via heuristics-used-in-practice (HUIP). If the scheduling process leads to a training company shortfall that cannot be corrected, then the training schedule and the corresponding training base scenario are *infeasible*. However, because training companies are aggregated across all installations in mid-term aggregate analysis, it is not possible to identify where (at which installations) the training company shortfalls occur. Therefore, the mid-term aggregate model is disaggregated into separate sub-models by installation and training program leading to near-term disaggregate analyses.

The HUIP scheduling methods of mid-term aggregate scheduling are also used for near-term disaggregate scheduling. By solving each of the disaggregated scheduling problems, it is possible to identify training resource shortfalls by installation and training week. Unfortunately, problem disaggregation causes the solution space to explode which necessitates reducing the size of the problem. Accordingly, the planning horizon for the near-term disaggregate problem is shortened to one year. Infeasible scenarios from long-, mid- and near-term analysis may be iteratively revised until they become feasible either by fixing the recruiting (i.e., training) objective and iteratively increasing training resource levels or, alternatively, fixing the training structure (training companies) and iteratively decreasing the recruiting objectives (that drive the training requirement) until a feasible scenario is found. These approaches are also followed in *ARMY*. However, when recruiting objectives are fixed, training company shortfalls (representing an

additional number of training companies required to meet the training objective) are automatically determined by the fully automated SPH and MPH scheduling procedures.

The 1989 DSS generates numerical and graphical output that is consistent in form and content with information used at the time to support training resource scheduling scenario analysis. The 1989 DSS partially automated some aspects of scheduling with heuristics-used-in-practice to substantially reduce the time to obtain a schedule from several days (or at best several hours) using manual methods, to approximately thirty minutes (implemented on a 286 microcomputer). An improved version of the 1989 DSS is still in use at TRADOC Headquarters today.

However, several severe drawbacks exist with both the original and improved versions of the 1989 DSS. First, resource scheduling remains a manual trial-and-error process which may lead to the incorrect classification of a feasible training base scenario as infeasible, thus eliminating a viable training scenario from further consideration. Second, the trial-and-error scheduling approach of HUIP can result in different scheduling solutions for the same training base scenario. Scheduling consistency and quality of results when scheduling with HUIP are highly dependent upon the skill and experience of the analyst. Finally, the system does not incorporate performance measures that support comparative analyses for appraising the quality of competing feasible training schedules.

## 5.2 System Development Process

A major effort has gone into designing, testing, and implementing a fully operational *Decision Support System for Army Basic Combat Training Resource Management per Year* to eliminate the shortcomings identified above, and to improve

support for decision making at the major levels of basic training management discussed previously. *ARMY* development followed four sequential, overlapping tasks.

Task 1. Functional Description of the *ARMY* System;

Task 2. Preliminary Design of the *ARMY* Architecture and System Modules;

Task 3. Development of the *ARMY* System Prototype;

Task 4. Full Development of the *ARMY* System.

The *ARMY* development process is shown in Figure 10, along with the subtasks accomplished for each major task.

The system development steps follow the decision analysis process for solving basic training problems to the extent possible. These steps include structuring the basic training problem, generating resource schedules for analyzing training scenarios, and representing system output in formats that support the decision process.

The main objective of Task 1 was to identify the primary functions of the *ARMY* system in terms of how the system could best support the decision process. In Task 2, system architecture was graphically represented through a set of interconnected modules where each module corresponds to a functional requirement of the system (see Figure 11). Module relationships were defined according to four attributes:

1. module input and output;
2. module functions;
3. functional procedures that define the logic and rules by which each module operates; and
4. flow of data between modules.

In Task 3, prototypes of each module were implemented within a common computer operating system (see below), and procedures were developed to control the flow of data between modules. The final step of Task 3 was prototype testing. In Task 4, the modules were linked to form the complete system. Task 4 concluded with full system testing.

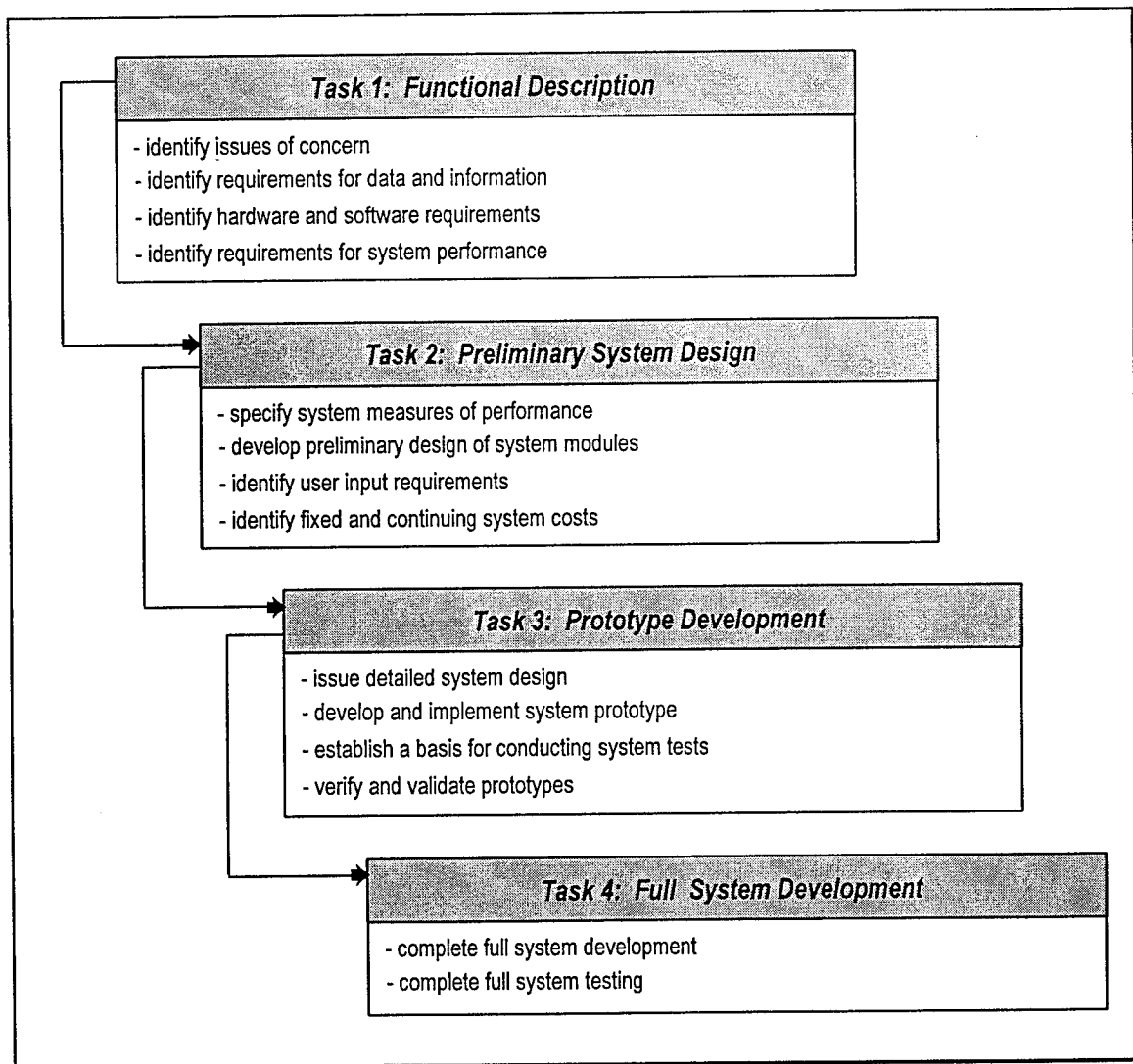


Figure 10. Decision Support System Development Phases

### 5.3 System Architecture

The *ARMY* system is based on a modular design and implemented in a computer spreadsheet environment called *LOTUS 1-2-3 for Windows, Release 4* that provides a fully integrated environment for model development. In the *LOTUS 1-2-3* spreadsheet, the system modules are dynamically linked (see Figure 11) to enable dynamic data exchange (DDE). The *LOTUS 1-2-3* spreadsheet software is ideally suited for handling thousands of calculations required to generate a weekly training resource schedule. Advanced macros provide programming flexibility for implementing and fully automating the scheduling and policy improvement routines, and for streamlining routine scheduling calculations. *LOTUS 1-2-3* also features a set of built-in statistical functions useful for analyzing resource scheduling output to support decision analysis. The *ARMY* system is centered around the dynamic system model of the Army's Basic Training Program presented in Chapter 2. The modular system design facilitates tailoring the *ARMY* to other military training programs as discussed previously. *ARMY* architecture and system modules are shown in Figure 11. The descriptive module names indicate the primary functionality of each *ARMY* module.

The *Resource Scheduler Module* forecasts the number of recruits to arrive for training each period and then computes the number of training companies required to start training each week. The module also schedules basic training companies (or other training resources) in each period of the planning horizon based on user-specified initial conditions and assumptions (see Sections 2.3 and 4.2).

The *Policy Improvement Module* invokes the single-pass heuristic to generate an initial feasible training resource schedule (if one exists) that is sequentially improved via the multi-pass heuristic. The logical flow of resource scheduling and heuristic policy improvement is diagrammed in Figures 5 through 9 of Chapter 4. Both the single- and

multi-pass heuristic scheduling algorithms are completely automated, thus eliminating the need for an analyst to interact with the computer model as is the case when scheduling with the heuristics-used-in-practice of the 1989 DSS.

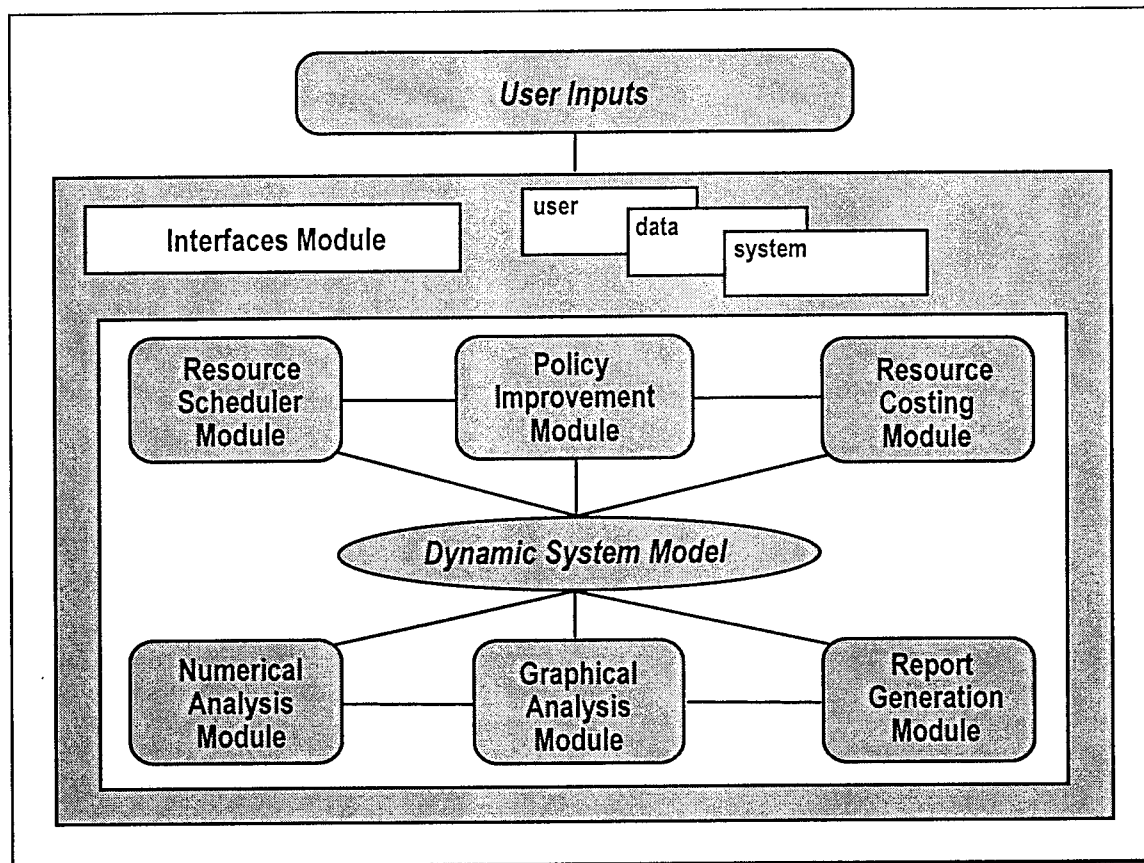


Figure 11. Decision Support System Architecture and Modules

The *Numerical Analysis Module* summarizes scheduling information and computes scheduling statistics. The *Graphical Analysis Module* provides graphical output of scheduling information and scheduling statistics. The *Report Generation Module* prints numerical and graphical results in formats tailored to support the decision process. All graphs may be displayed to the computer screen or printed, either from the keyboard or by "clicking" on user-friendly "buttons" using a mouse pad. Results in



Chapter 6 gives examples of numerical and graphical output. Output from an illustrative scheduling session with the *ARMY* system is found in Appendix C.

#### 5.4 Resource Costing Module

The Resource Costing Module estimates various training program costs from the training resource schedules generated via the single- and multi-pass heuristics of the *ARMY* system. The idea of using training resource schedules to generate training program cost estimates was provided by Rao [11] who developed a manpower planning model analogous to Wagner and Whitin's ELSP model. Rao's model uses manpower requirements for future periods to minimize manpower system costs for a number of fixed and variable recruitment costs.

Training program resource costs are incorporated into *ARMY* to make the DSS more supportive of training program decision making. Cost measures currently estimated by *ARMY* include the following.

- Total annual costs for the two-year planning horizon including fixed costs, variable costs, costs by resource, and total program cost.
- Year-by-year cost differences and percent yearly changes in training costs reflecting training installation changes (attributable to training base scenarios) or force structure changes (e.g., recruiting targets). Yearly cost differences are computed for each type of cost given above.
- Average variable cost are determined per training cycle for the by training program, by training battalion, by training company, and by trainee.
- Average variable costs are computed per training company (per training cycle) by resource.

- Average program cost per recruit, and average program cost per recruit by training resource are also estimated.

Typical cost data for this study was provided by the Directorate of Resource Management<sup>2</sup> (DRM) of Fort Benning, Georgia. The data is based on a 1993 study of basic training resource utilization conducted by the Fort Benning DRM. The *1993 Fort Benning Study* measured fourteen training resource costs for a single basic training battalion consisting of one headquarters company and five basic training companies during six consecutive training cycles. Ten representative cost items from the Fort Benning study were selected to illustrate the costing methodology. These cost items are: (1) in-processing activities, (2) basic training support, (3) supply operations, (4) maintenance of basic training equipment and materials, (5) transportation services, (6) laundry services, (7) food services, (8) personnel support, (9) ammunition for weapons qualification, and (10) utilities. The four cost items not used are housing costs for linen replacement, engineer support for refuse collection, copying and printing costs for records management, and manpower costs for maintenance of real property.

Training costs are classified (for this study) according to four categories; direct-fixed, direct-variable, indirect-fixed, and indirect-variable. *Direct costs* account for expenditures directly related to recruit in-processing and basic training. *Indirect costs* represent the proportion of *base operations* costs attributable to initial entry training. As shown in Figure 7, base operations costs may be further broken down into two categories: *base support* services and *facility support* services. Appendix D gives a partial listing of base operations costs by category (for reference), however, it is important to note that indirect training costs are likely to vary across training installations, and in some cases,

---

<sup>2</sup>Courtesy of Major Scott Manderville, then of the Directorate of Operations and Training (DOT), Fort Benning, GA.

may not apply at all. Only one indirect cost, utilities, was reported in the 1993 Fort Benning Study. Numerous attempts were made by the author to obtain additional indirect costs from other training installations and TRADOC Headquarters as well, however, none of the agencies were able to provide actual (or estimated) indirect costs for the initial entry training program.

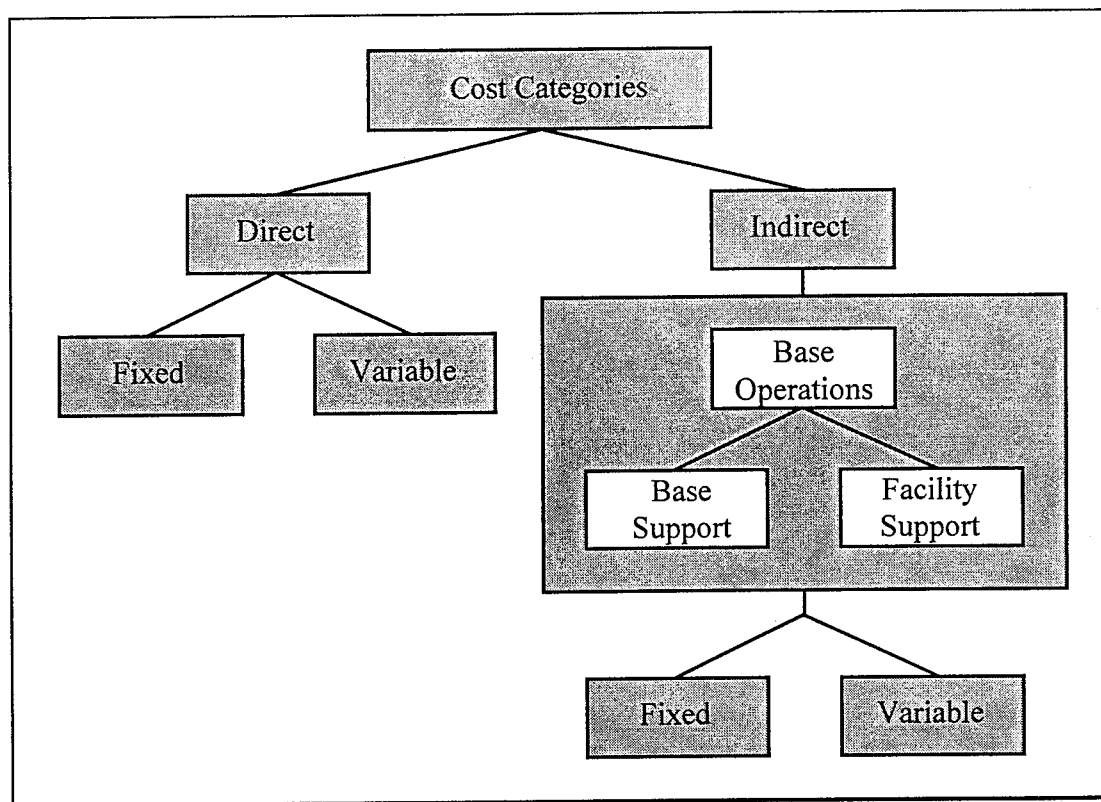


Figure 12. Categories of Training Resource Costs

The 1993 Fort Benning Study reports summarized annual fixed and variable costs for the resources listed above. Therefore, in order to estimate resource costs for training resource schedules from the *ARMY* system, it was necessary to break down the annual fixed and variable costs into cost factors. Training resource cost factors for each cost

category are given in Table 1. Details of cost factor computations are provided in Appendix E.

Cost Factors	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Personnel In-Processing	\$27,000				\$85
Basic Training Support	\$83,000			\$2166	\$345
Supply Operations			\$61,000		
Maintenance of Materials			\$68,000		\$290
Transportation Services		\$2500	\$56,000	\$1833	
Laundry Services					\$145
Food Services					\$788
Personnel Support			\$26,000		
Ammunition				\$131	\$388
Utilities:		\$55,500			
<b>GRAND TOTAL:</b>	\$110,000	\$58,000	\$211,000	\$4130	\$2041

Table 1. Training Cost Factors for Basic Training Analysis

Fixed costs are generally assessed at the beginning of each year (YR) of the planning horizon based on either the number of training battalions or the number of training companies, available at the beginning of each year. Both fixed and variable costs are computed by training battalion (BN) and by training company (CO). A variable cost per recruit, by resource, is also determined.

It is important to note that the cost factors of Table 1 are based entirely on Fort Benning training costs, and do not reflect regional or seasonal cost differences known to exist from installation-to-installation. Therefore, the *aggregated* experimental results of Chapter 6 are only meant to illustrate how cost factors can be used to estimate basic training resource costs from the training resource schedules generated via the heuristic procedures of Chapter 4.

*Recruit In-Processing*

Recruit in-processing costs include costs associated with in-processing new recruits during the fill week. The 1993 Fort Benning Study include the annual salary for one civilian employee responsible for soldier in-processing activities, and the cost of general supplies and organizational clothing issued to recruits.

*Basic Training Support*

Basic training support activities include (1) the fixed cost of annual salaries for two civilian personnel providing administrative support to training companies, and (2) variable costs for soldier issue items, company issue items, issue of general supplies, cleaning kits for individual and crew-served weapons, load bearing equipment (LBE), and issue of medical supplies.

*Supply Operations*

Supply operations costs include three fixed costs per training battalion for the annual salaries of one sewing machine operator to alter clothing issued to new recruits, one tailor to measure recruits for clothing issue, and one clerk to transcribe data and process trainee records.

*Maintenance of Materials*

Maintenance costs from the 1993 Fort Benning Study include annual salaries for maintenance workers, annual maintenance costs for direct support (DS) and general support (GS) maintenance of weapons and load bearing equipment (LBE), and the annual cost of repair parts for weapons and LBE.

### *Transportation Services*

Transportation costs pertain to both administrative and training company transportation requirements. The 1993 Fort Benning Study reports annual transportation costs by type of vehicle computed as the operating cost per mile, times the average miles operated per training cycle, times the number of vehicles operated (by type). The cost module calculates an administrative transportation cost per training battalion for each set of five basic training companies that start a training cycle, along with training company transportation costs per training cycle. An annual cost for drivers to support training companies is also included.

### *Laundry Services*

Weekly laundry service is provided to recruits during initial entry training. The Fort Benning Study gives an annual cost for laundry service provided to recruits based on the six basic training cycles assuming 200 recruits per training company. The laundry cost factor represents the laundry cost per trainee per training cycle. The cost of operating and maintaining the laundry facility is not included.

### *Food Services*

The 1993 Fort Benning Study reports an estimated annual cost for operating the battalion dining facility. For the cost module, the annual food service cost is broken down using the DRM's assumptions of 200 recruits per company and six basic training cycles per year to obtain a food service cost per recruit.

### *Personnel Support*

Personnel support includes the annual salary paid to one civilian employee per training company responsible for providing administrative support to each basic training company.

### *Ammunition*

Two types of ammunition are included in the study: (1) ammunition for the individual assault weapon (M16); and (2) ammunition for the squad assault weapon (SAW). Ammunition cost factors are computed for a fixed amount of ammunition (called the *basic load*) issued to training company cadre for instruction purposes, and a variable ammunition cost per recruit for weapons training and qualification.

### *Utilities*

Utilities is the only indirect cost reported in the 1993 Fort Benning Study. Utilities includes annual costs for electricity, natural gas, water services, and sewer services for the training battalion. The utilities cost reported are average cost based on several years of historical data.

## 5.5 Benefits of the DSS

*ARMY* is a user-friendly computer software package that can support analysis of many practical problems encountered in basic training management. Experiments with *ARMY* have demonstrated its capability to evaluate different scenarios for downsizing or realigning basic training installations, changes to recruitment objectives, and scenarios that build-up the training base structure as well. *ARMY* can be used to support the decision making process by generating practical resource scheduling performance measures that help decision makers select a "best" alternative from a set of feasible ones.

Finally, *ARMY* quickly generates realistic training resource schedules that include training resource cost estimates based on resource utilization.



## 6. RESULTS

The size of the real-world basic training problem precludes implementation of an exact solution method (see Sections 3.1 and 3.4), which could be used as a yardstick to measure the effectiveness of the heuristic scheduler. However, modifications to assumptions and constraints given in Chapter 2 that further constrain the subsets of feasible training resource decision elements (e.g., admissible actions for company strengths and training cycles) given by  $\mathcal{X}_j[t, I_j(t)] \subset \Omega$  and  $\mathcal{Y}_j[t, I_j(t)] \subset \Lambda$  (see Chapter 3), make it possible to implement the dynamic programming technique for a much simplified version of the basic training problem. Section 6.2 compares, for twelve example problems, optimal scheduling solutions using DP with results from the heuristic scheduling procedures of Chapter 4.

First, however, we follow an approach similar to that used by Yang and Ignizio [13] to evaluate their heuristic method for solving the battalion training problem discussed in Chapter 2, where an optimal solution to their very large real-world problem could not be obtained. Yang and Ignizio evaluated their heuristic method against two other heuristics for three different scheduling problems. Results were compared using two performance measures: (1) makespan of the schedule, and (2) CPU (central processing unit) time to generate the schedule.

We compare the three heuristic scheduling methods of Chapter 4

- Heuristics-Used-In-Practice (HUIP);
- Single-Pass Heuristic (SPH);
- Multi-Pass Heuristic (MPH).

using four performance measures for twelve realistic test scenarios.

### 6.1 Heuristic Scheduling Results

The test scenarios for this study are based on 1988 training base structure and illustrative recruiting targets for 1989 and 1990. The number of training companies deactivated ( $D_j$ ) by year  $j$  and scenario are shown in Table 2.

Scenario:	1	2	3	4	5	6	7	8	9	10	11	12
$D_{1989}$ :	5	10	15	20	0	0	0	0	5	10	15	10
$D_{1990}$ :	0	0	0	0	5	10	20	30	5	10	15	10

Table 2. Training Company Deactivation Scenarios

The annual recruiting targets for this study are  $R_{1989} = 136,000$  and  $R_{1990} = 128,000$ . Initially, 130 training companies are available for training recruits at the beginning of 1989 (year 1). Training cycle lengths are initialized at ten weeks. For all scenarios, except 10 and 12, training company strengths  $x_j(t)$  are initialized at 200 recruits per training company for the HUIP method and 150 recruits for the SPH and MPH methods. For Scenarios 10 and 12, all three heuristics are initialized at 150 and 200 recruits per company, respectively. Initial conditions for company strengths, by scheduling method, are summarized in Table 3.

Method / Scenario	1-9, 11	10	12
HUIP:	200	150	200
SPH:	150	150	200
MPH:	150	150	200

Table 3. Initial Company Strength Values

Experimentation with the SPH and MPH methods reveals that processing time and schedule quality (as measured by the instructor-to-student ratio) are highly dependent

upon the step size  $n$  of the Company Strength Policy Improvement Algorithm (see Section 4.2). A step size of  $n = 5$  was chosen for this study based on experiments with step size increments from one to ten, to evaluate tradeoffs between processing time and schedule quality. Other step sizes can be easily accommodated in the model.

### *Performance Measures*

The "goodness" of any training resource scheduler may be evaluated according to different performance measures, such as (1) the "quality" of the training schedule obtained (e.g., the instructor-to-student ratio), (2) training program cost, (3) training resource utilization as measured by average (over the planning horizon or by year) of idle training companies, training company strength, or the number of training companies to begin a training cycle each period, and (4) CPU time. For this study, training resource schedules and scheduling methods are evaluated according to following four performance measures:

1. processing time - as the time needed to obtain a feasible training resource schedule;
2. the training "quality" performance measure,  $\sum_{t=1}^{T_J} \frac{1}{x_j(t)}$  (instructor-to-student ratio), where we assume, for simplicity, there is only one instructor per training company;
3. the average number of idle training companies per period,  $\frac{1}{T_J} \sum_{t=1}^{T_J} I_j(t)$  - a measure of training resource utilization; and

4. basic training resource costs - computed using the cost factors described in Section 5.4.

### *Scheduling Results*

All computational tests were conducted by the author on a 486/D66 microcomputer. The user of the computer-supported HUIP method must interact with the computer model to manually determine the company strength and cycle length decisions to take in each period. These decisions are determined automatically via the SPH and MPH methods.

Only training base scenarios where a feasible training resource schedule exists were considered to ensure a viable comparison across test scenarios and scheduling methods. It is worth noting that the author's first attempt to find a feasible training resource schedule for Scenario 8 was unsuccessful using HUIP. If a feasible training resource schedule for this scenario was not obtained using the SPH method, Scenario 8 might have been incorrectly classified as *infeasible*.

Table 4 (see below) gives numerical results by training base scenario (*Scen*) and scheduling method (*HUIP*, *SPH*, *MPH*) for performance measures 1, 2 and 3. Results for training program costs are given separately in the succeeding subsection *Comparing Training Program Costs*. Columns  $D_1$  and  $D_2$  of Table 4 correspond to rows  $D_{1989}$  and  $D_{1990}$  of Table 1, respectively; each entry indicates the number of training companies to deactivate each year. The columns *Time*, *Ratio* and *Idle Co* correspond to performance measures 1, 2 and 3, respectively.

Processing *Time* of the heuristic, measured manually with a stop watch, represents the time in minutes and seconds required to generate a training resource schedule. MPH

processing time represents time required for the MPH to improve the initial feasible training resource schedule of the SPH.

*Ratio* values in Table 4 represent the instructor-to-student ratios summed over the planning horizon  $T_j$ . The *utopian* value for this performance measure is

$$\sum_{i=1}^{96} \frac{1}{150} = 0.64; \text{ so named because it will not be attainable, in general.}$$

*Idle Co* represents the number of idle training companies averaged over the planning horizon (rounded to the nearest whole training company).

				HUIP			SPH			MPH <sup>1</sup>	
Scen	$D_1$	$D_2$	Time	Ratio	Idle Co	Time	Ratio	Idle Co	Time	Ratio	Idle Co
1	5	0	2:57	.478	34	:53	.581	8	:36	.582	8
2	10	0	3:53	.476	30	:58	.568	6	:38	.569	6
3	15	0	5:53	.473	25	1:01	.554	5	:38	.556	5
4	20	0	9:52	.469	21	1:06	.540	4	:47	.542	4
5	0	5	3:03	.480	36	:51	.585	9	:36	.587	9
6	0	10	3:21	.478	34	:53	.577	8	:39	.578	8
7	0	20	3:23	.471	31	:57	.559	7	:44	.561	7
8	0	30	9:57	.464	28	1:05	.542	7	:48	.544	7
9	5	5	5:09	.477	32	:56	.572	7	:42	.574	7
10	10	10	10:27	.515	8	1:05	.549	6	:49	.551	6
11	15	15	11:48	.465	20	1:13	.527	4	:55	.529	4
12	10	10	4:51	.472	25	:41	.470	22	11:54	.549	7

Table 4. Numerical Results of Scheduling Heuristics

<sup>1</sup>Processing time for the MPH; does not include processing time to obtain an *initial feasible resource schedule* needed as the starting point for the MPH (see Section 4.3). For this study, the initial schedule for the MPH was generated using the SPH.

### Comparing Processing Times

Figure 13 graphs processing time for the three heuristics. Observe that processing times increase as  $D_j$  increases. However, the increase in HUIP processing time by scenario is much more dramatic. Results for Scenarios 1 through 11 show that the SPH finds schedules 6.4 times faster than the HUIP method, and 1.7 times faster than the MPH if the time to find an initial feasible schedule (via SPH) is added to MPH processing time. Similarly, the MPH is 3.7 times faster than the HUIP method.

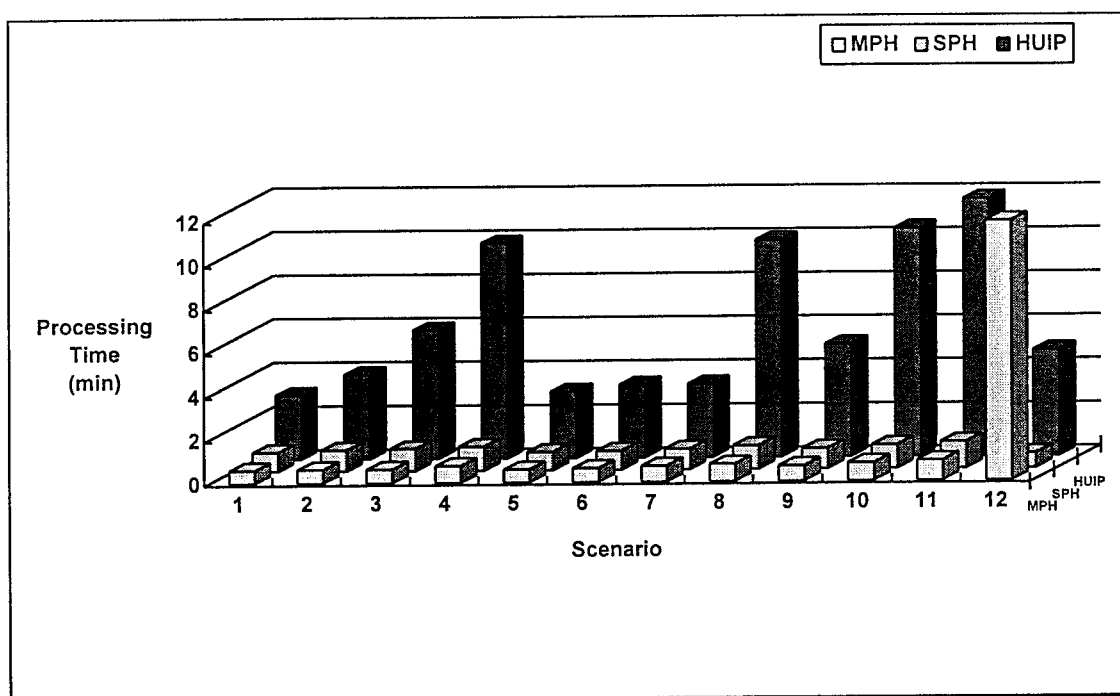


Figure 13. Comparison of Heuristic Processing Times

For Scenario 12, company strengths are initialized at 200 recruits per training company which dramatically reduces SPH processing time, but results in a significantly "poorer" initial feasible schedule for the SPH (see Table 4) which negatively affects MPH processing time performance, since the MPH uses the SPH schedule as its starting point (see Section 4.3). Consequently, many more iterations of the MPH's Company Strength

Policy Improvement Algorithm are necessary to "tighten" the solution in each period causing MPH processing time to increase.

### *Comparing the Instructor-to-Student Ratios*

As discussed previously, the instructor-to-student ratio serves as a performance measure of training quality (see Chapter 2). Assuming one instructor per training company, the ratio is the inverse of company strength decisions in each period. Therefore, the problem is suboptimized by making company strengths as small as possible. Figure 14 compares instructor-to-student ratios for each heuristic procedure, where the *utopian* value of the performance measure is 0.64.

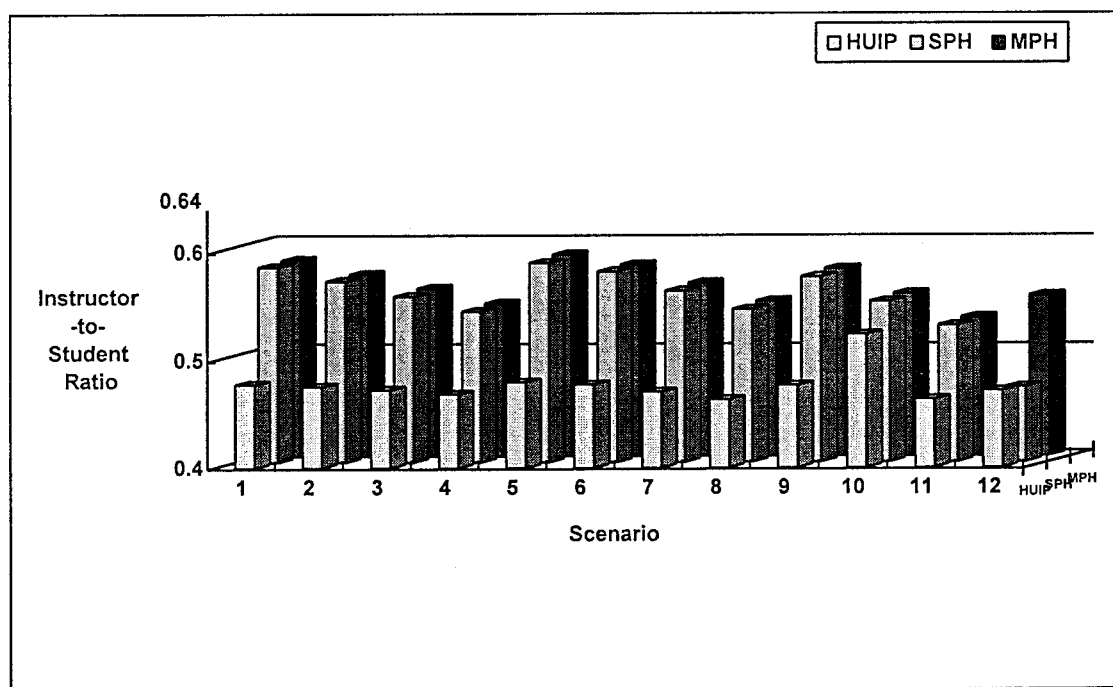


Figure 14. Comparison of Instructor-to-Student Ratios

On average, the quality of the MPH and SPH schedules were 19% and 18.7% better than the HUIP schedules, respectively. The SPH and MPH generate higher quality

schedules for all cases except Scenario 12, where the HUIP marginally outperforms the SPH. The reason for this is that the SPH attempts to find a feasible solution by increasing training company strengths one step at a time from the current company strength value, but only in those weeks where there is a training company shortfall ( $I_j(t) < 0$ ). In Scenario 12, initializing company strengths at 200 recruits eliminates a number of training company shortfalls that would have otherwise occurred had company strengths been initialized at 150 recruits per company. Thus, the initial condition imposes a "penalty" on the quality of the solution that can be attained via the SPH for Scenario 12. However, Scenario 12 also illustrates the how the MPH methodically tightens the company strengths in each period of the planning horizon, thereby improving the objective function value, from 0.470 for the SPH to 0.549 for the MPH; a substantial 16.8% improvement in quality. For Scenarios 1 through 11, the MPH gives an average improvement over the SPH solutions of only three tenths of one percent (0.3%), but as shown in Figure 13, the processing time for the MPH is so small (in general) that the small improvement in the "quality" measure may be worth getting. Even though this modest gain may seem inconsequential, it may be quite significant in terms of total dollars when applied to annual training program costs.

In Figure 15 (see below), the scheduling results are measured against the *utopian* value for scheduling performance. The scheduling results are expressed as percent of *utopian* value, here 100 percent represents an instructor-to-student ratio of 0.64. From Figure 15, it is evident that the "effectiveness" of the result depends, in part, upon the number of training companies deactivated in the corresponding scenario.



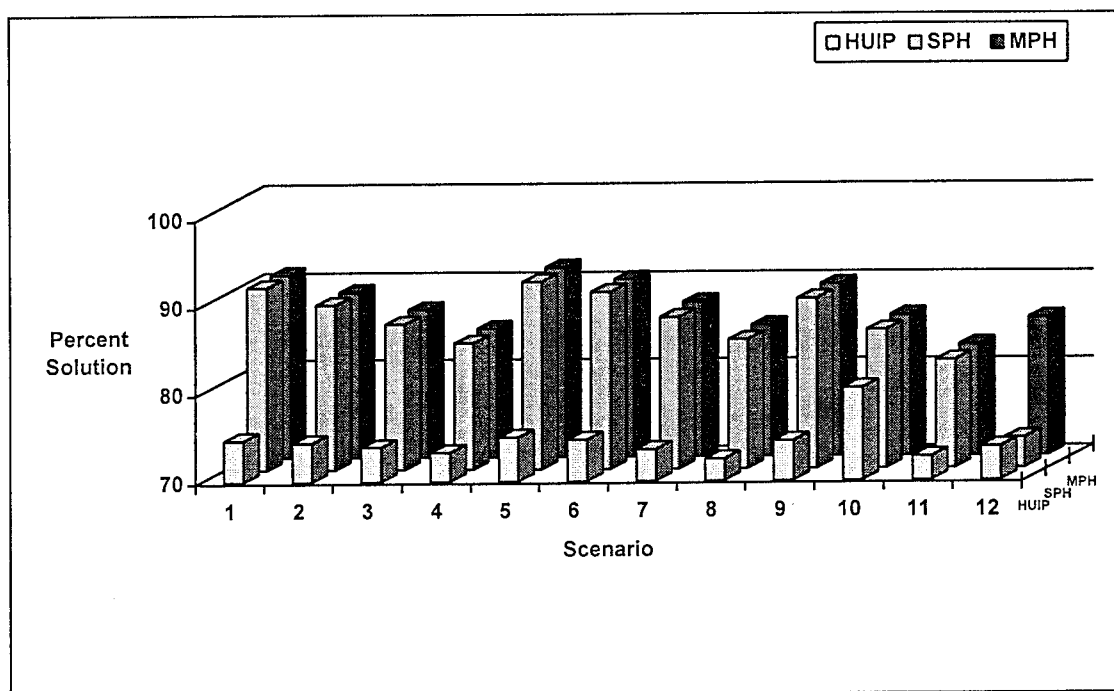


Figure 15. Effectiveness of Heuristic Solutions as a Percent of the *Utopian*

The various scenarios selected for this study represent a "balanced" set of problems (that is, balanced according to the distribution of the severity of training company cuts) for testing the scheduling procedures. The problems represent a range of reasonable conditions so as to not (overly) bias the estimates of scheduling effectiveness. Excluding the special cases, Scenarios 10 and 12, the HUIP, SPH, and MPH methods generate solutions that are (on average) approximately 74%, 87.5%, and 87.8% of the *utopian* value of the "quality" performance measure, respectively. It is believed that if the SPH and MPH methods are used (or tested) under similar conditions in the future, then the average performance of the SPH and MPH methods, under these conditions, will be approximately 87% of the *utopian* solution; substantially better than the estimated 74% for the HUIP method.

### *Comparing the Average Number of Idle Training Companies*

The third performance measure, average number of idle training companies, evaluates training resource schedules in terms of training company utilization. Note that the objective of finding the maximum instructor-to-student ratio (by making company strengths as small as possible in each period), coincidentally, drives idle training companies (or other idle economic resources) to minimal levels. *ARMY* supports basic training decision making by generating useful experimental data, such as the average number of idle training companies, that helps to determine whether or not the training base is either over- or under-structured relative to projected recruiting targets. Figure 16 compares idle training companies for the various training base scenarios.

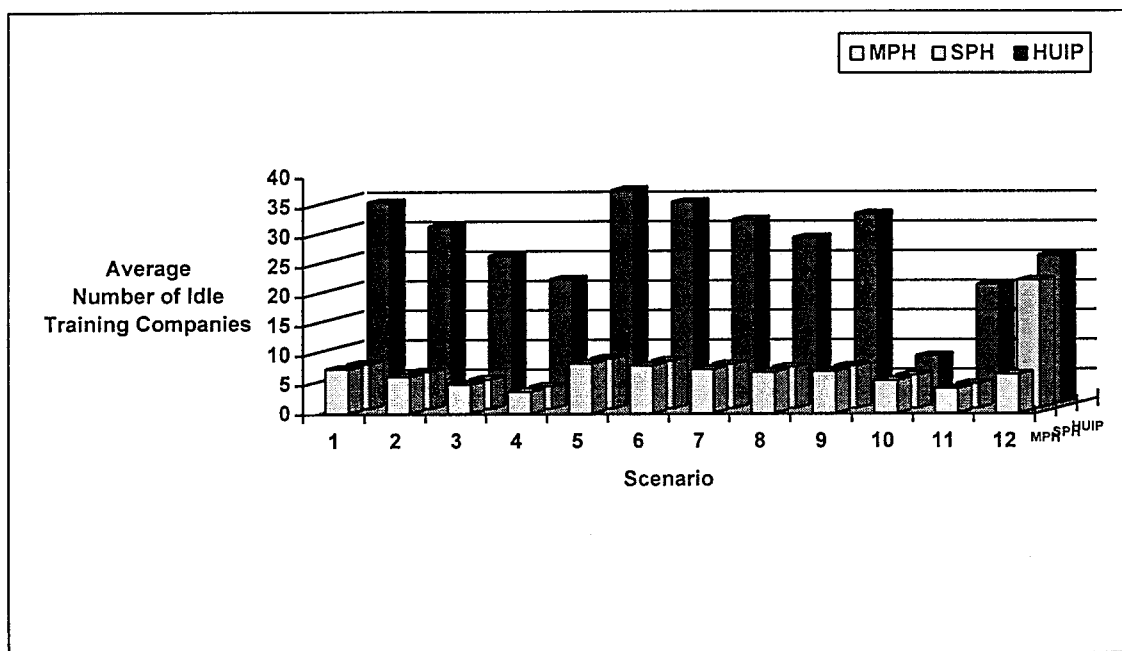


Figure 16. Comparison of Idle Training Companies

Figure 16 reveals that for Scenarios 1 through 8, the average number of idle training companies declines with training company deactivations (as expected): when

fewer companies are available to train recruits, then a higher proportion of the companies still available must be pressed into use.

Next, observe that for all scenarios except 12, idle training companies for the SPH and MPH are exactly equal. This is because the SPH Company Strength Policy Improvement Algorithm is designed to satisfy the scheduling feasibility constraint  $(I_j(t) \geq 0)$  by increasing company strengths in those weeks where  $I_j(t) < 0$  in steps of  $n$  until  $I_j(t) = 0$ . The instant when the feasibility constraint becomes tight (i.e., when equality holds), the algorithm stops. Therefore, although the MPH is able to tighten company strengths in each period when company strengths are initialized at the lower bound, as with the SPH, the MPH cannot further reduce idle training companies because the SPH makes these constraints tight in each period. However, when training companies are initialized at a value greater than the lower bound, such as at 200 recruits as in Scenario 12, then the condition  $I_j(t) > 0$  may occur (in many periods). Then MPH's refinement of the company strengths leads to a reduction of idle training companies as well (see results for Scenario 12). Differences in idle companies between the SPH/MPH and the HUIP methods (excluding Scenario 12) shows that the SPH/MPH are 3.8 times more effective, on average, at reducing the number of idle training companies than the HUIP method.

### *Comparing Training Costs*

The training costs presented here are based on the ten cost items listed in Section 5.4. These analyses illustrate how cost factors might be used to estimate training resource costs for schedules produced via the *ARMY* system. Although these cost estimates are only illustrative, it is believed that incorporating additional training program costs into

the system will lead to realistic training resource cost estimates that support both long-range planning and current operations.

<b>1989 Costs for Scenario 11</b>	<b>SPH</b>	<b>HUIP</b>
Total Program Cost	\$286,896,573	\$285,764,013
Total Fixed Cost (BN & CO)	\$26,795,000	\$26,795,000
Total Variable Cost (BN, CO, & Recruit)	\$260,101,573	\$258,969,013
Average Variable Cost Per Training Cycle	\$384,745	\$432,342
Average Program Cost Per Recruit	\$2,344	\$2,335
<b>1990 Costs for Scenario 11</b>	<b>SPH</b>	<b>HUIP</b>
Total Program Cost	\$267,794,283	\$267,070,703
Total Fixed Cost (BN & CO)	\$23,300,000	\$23,300,000
Total Variable Cost (BN, CO, & Recruit)	\$244,494,283	\$243,770,703
Average Variable Cost Per Training Cycle	\$397,135	\$430,349
Average Program Cost Per Recruit	\$2,325	\$2,318

Table 5. Summary of Training Resource Costs

Table 5 summarizes training program costs for training resource schedules obtained with HUIP and SPH for Scenario 11 (see Appendix F for additional cost estimates).

The *ARMY* system includes options for viewing costs estimates graphically. Figure 17 compares the program cost estimates for the schedules generated using the SPH and HUIP methods for all of the twelve "1989" training base scenarios.

These results show that the costs for the HUIP method are consistently lower than the total program costs of the SPH method. This should be expected since SPH generates higher "quality" schedules than the HUIP procedure and higher quality training costs more. This result is due, primarily, to the fewer training starts required by the HUIP

method because of it initializes company strengths at 200 recruits. The differences in the way the SPH and HUIP schedules are obtained may also significantly impact training program execution, training program funding, and training resource allocation.

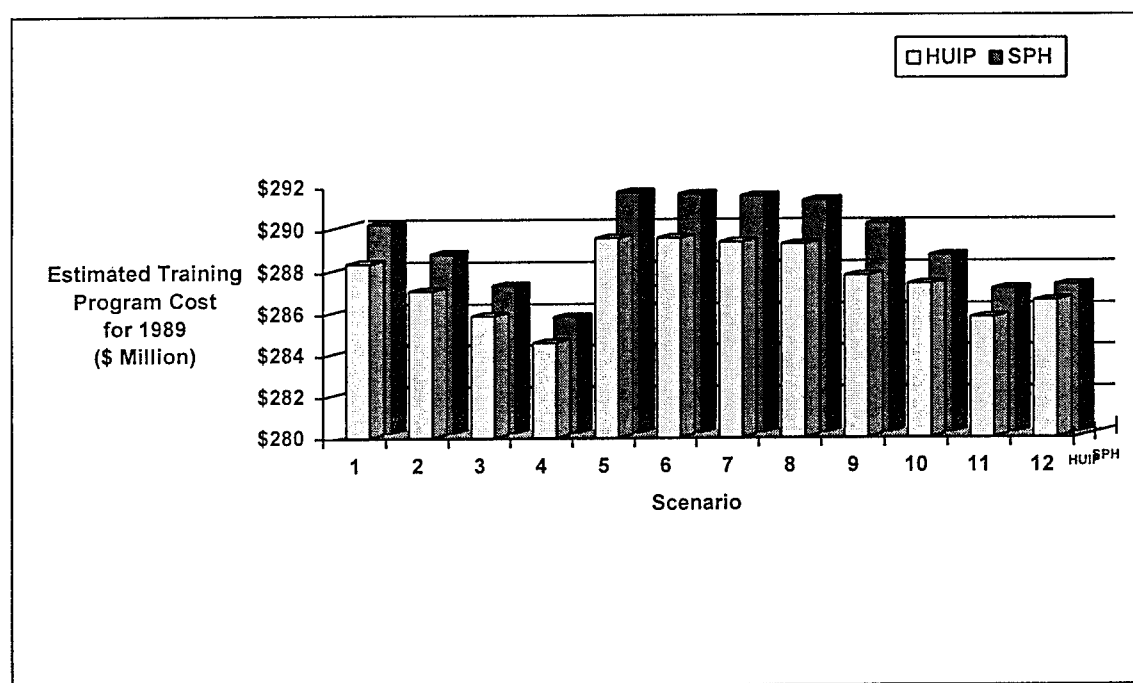


Figure 17. Comparison of Total Training Program Cost by Scenario

## 6.2 Comparison of Optimal and Heuristic Scheduling Results

In this section, we compare the SPH-MPH<sup>2</sup> and DP scheduling methods for solving a simplified problem to observe the differences in the quality of optimal versus heuristic solutions. As explained in Section 3.1, the state space for one stage of a real-world basic training problem may contain  $3 \times 130 \times (101)^9$  (or approximately  $4.27 \times 10^{20}$ ) possible states. The high dimensionality of the real-world problem makes it impractical to implement an exact solution procedure such as dynamic programming. However, it is

<sup>2</sup>SPH-MPH notation denotes that the SPH is used to generate the *initial feasible training resource schedule* used as the starting point for the MPH.

possible to use dynamic programming to solve a simplified version of the problem to make a comparison of heuristic versus DP results. Accordingly, we make the following modifications in characterizing the problem scenario:

- training cycle length decisions are removed from consideration by fixing cycle lengths at  $y_j(t) = 2$  throughout the planning horizon (i.e., the time lag in the dynamics is reduced from ten periods to one period);
- the maximum allowable number of idle training companies is reduced from 130 to 20 (this bound was determined by experimentation with SPH);
- allowable (feasible) company strength decisions are reduced from 101 (for a step of 1) to 21 (corresponding to a step size of 5).

These simplifications reduce the state space (for one period) by seventeen orders of magnitude: from  $4.27 \times 10^{20}$  possible states for the real-world problem to 9,261 ( $21 \times 21 \times 21$ ) possible states for the current problem. The planning horizon is reduced from 96 to 48 weeks<sup>3</sup>, thus establishing a new *utopian* upper bound of 0.32 for the "quality" performance measure. Although reducing the number of stages does not reduce the state space, it does lower the requirement for computer memory and reduces the CPU time for generating an optimal solution.

#### *Implementation of the DP Algorithm*

The state transition equation for a one-period lag problem is given by

$$I(t+1) = I(t) + \frac{r(t-1)}{x(t-1)} - \frac{r(t)}{x(t)},$$

---

<sup>3</sup>Note the subscript  $j$  denoting the year of the planning horizon can be dropped since 48 weeks correspond to a one-year planning horizon.

where company strengths  $x(t)$  are restricted to the allowable set of integer-valued decisions defined in Section 3.2 as  $\mathcal{X}[t, I(t)] \subset \Omega$ . This may seem to be a "trivial" problem that has little (or no) practical value (to basic training management) as far as real-world decision making is concerned. However, implementation of the DP algorithm for the one-period time lag problem is an important first step towards solving larger, more complex problems. The steps necessary to implement the DP algorithm to solve a larger problem are essentially the same as those required to solve the one-period lag problem. The major differences between the one-period time lag problem and those with longer time lags are (1) the exponential explosion in the size of the problem (see Table 6) that comes with state augmentation, and (2) an increase in the complexity of the reformulated problem due to additional terms in the state transition equation. Nevertheless, the initial effort on implementing the exact DP approach has provided very good insight into, and an appreciation for, issues that will become more important in future efforts to solve larger problems. Such issues include the practical aspects of managing computer memory effectively, writing efficient computer codes for DP algorithms, and the choice of software and hardware for implementing and running the programs. Table 6 shows how quickly the size of the state space increases as time-lag in the dynamics increases for a company strength step size of five, and increases of three idle training companies per period for each incremental increase in time lag. Note that  $I \leq 48$  for the ten-period time lag in Table 6 compares favorably with the constraint  $I \leq 50$  for idle training companies given previously in Section 3.5.

Time Lag ( $l$ )	$x(t-l)$	$I(t)$	$x(t)$	State Space
1	21	21	21	9,261
2	$21^2$	24	21	222,264
3	$21^3$	27	21	$5.2 \times 10^6$
4	$21^4$	30	21	$1.2 \times 10^8$
5	$21^5$	33	21	$2.8 \times 10^9$
6	$21^6$	36	21	$6.5 \times 10^{10}$
7	$21^7$	39	21	$1.5 \times 10^{12}$
8	$21^8$	42	21	$3.3 \times 10^{13}$
9	$21^9$	45	21	$7.5 \times 10^{14}$
10	$21^{10}$	48	21	$1.7 \times 10^{16}$

Table 6. Increase in the Size of the State Space for each One-Period Increase in the Time Lag of the Dynamic Model

DP implementation developed here was based on a backward recursion, although, since the problem is completely deterministic, a forward DP algorithm should work just as well (see Bertsekas [3], pg. 31). The planning horizon of 48 weeks starts in week one and ends in week 48. The augmented state for the one-period time lag problem is represented by  $[I(t), x(t-1)]$ . For each week  $t$  of the one-period time lag problem, there are 441 (e.g.,  $21 \times 21$ ) objective function values, denoted  $J^*[t, I(t), x(t-1)]$ , that must be determined to complete the matrix of optimal objective function values, denoted by  $J^*[t, *, *]$ . Each one of these  $J^*[t, *, *]$  values is determined by an enumeration of objective function values for each element of the  $1 \times 21$  array of allowable company strengths, denoted by  $X[x(t)]$ , and the *maximum*  $J[t, *]$  value is chosen from among the twenty one cases.

Objective function values are computed recursively back in time starting in period 48 and ending in period 1. The  $J^*[t, I(t), x(t-1)]$  array of optimal objective function values for the planning horizon is "built up" period-by-period so that by week 1 each



value  $J^*[1,*,*]$  of  $\mathbf{J}^*[1, I(1), x(0)]$  represents the optimal objective function value for the planning horizon for initial conditions of  $I(1)$  idle companies and  $x(0)$  company strength.

Two optimal objective function value arrays are needed for each period: (1)  $\mathbf{J}^*[t, I(t), x(t-1)]$  to store the  $J^*[t,*,*]$  values computed for the current period  $t$ , and (2)  $\mathbf{J}^*[t+1, I(t+1), x(t)]$  containing the values  $J^*[t+1,*,*]$  computed from the previous step (i.e., for the next period  $t+1$ ) that are needed to compute  $J^*[t,*,*]$  in period  $t$  (for details see step Week  $t$  below). Once all the  $J^*[t,*,*]$  values are computed for  $\mathbf{J}^*[t,*,*]$ , the values in  $\mathbf{J}^*[t+1,*,*]$  may be discarded and replaced by the elements of the  $\mathbf{J}^*[t,*,*]$  array that were just computed. This last operation serves as the preliminary step for working backwards for one period in the planning horizon.

If we assume a one-period "quality" measure value of zero for period 49, then the notation for computing objective function values  $J^*[t, I(t), x(t-1)]$  beginning with week 49 is as follows:

Week 49:

$$J[49, I(49), x(48)] = 0.$$

Week 48:

$$J[48, I(48), x(47)] = \max_{x(48) \in X[48, I(48), x(47)]} \left\{ \frac{1}{x(48)} + J[49, I(49), x(48)] \right\},$$

and continuing in a similar fashion,

Week  $t$ :

$$J[t, I(t), x(t-1)] = \max_{x(t) \in X[t, I(t), x(t-1)]} \left\{ \frac{1}{x(t)} + J[t+1, I(t+1), x(t)] \right\},$$

for  $t = 1, \dots, 47$ .

$\frac{1}{x(t)}$  represents the one stage "reward" for week  $t$ . The *Week  $t$*  performance function denotes the general form for computing the optimal "reward-to-go" from the current stage  $t$  to last stage  $T$ .  $I(t+1)$  is computed from the state transition equation for each element of  $X[x(t)]$ , given the state  $[t, I(t+1), x(t)]$  and recruit arrivals  $r(t)$  and  $r(t-1)$ . The optimal company strength decision  $x^*(t)$  is determined at the same time the optimal objective function value  $J^*[t, *, *]$  is determined; it is the value that *maximizes* the right hand side of each equation shown in steps *Week 49* to *Week 0*. If there is a tie between two or more  $J^*[t, *, *]$  values, ties are broken by selecting the  $J^*[t, *, *]$  that corresponds to the smallest company strength decision  $x^*(t)$  which is consistent with our objective of maximizing training "quality."

The DP algorithm was implemented as follows:

STEP 0. *Initialize the Problem.*

Create: one optimal decision array  $X^*[x(t)]$  for each period  $t = 1, \dots, 48$ .

Create: two "reward-to-go" arrays:  $J^*[t+1, I(t+1), x(t)]$ ,  $J^*[t, I(t), x(t-1)]$ .

Initialize:  $J[49, I(49), x(48)] = [0]$ .

Initialize: the 1x49 array of recruit arrivals;  $RA[r(t)]$ .

Initialize: the 1x21 array of allowable company strengths;  $X[x(t)]$ .

STEP 1. Implement the DP Recursion.

Begin: in Stage 48 and work BACKWARD in time to Stage 1,

Do: Tasks (1.1) through (1.8) for each of the 48 stages.

1. Do: the following for each state  $[t, I(t+1), x(t)]$  of  $J^*[t, I(t), x(t-1)]$ .
2. Compute:  $I(t+1)$  for each element of  $\mathbf{X}[x(t)]$  given  $[t, I(t), x(t-1)]$ ,  $r(t)$  and  $r(t-1)$  according to the following two cases (see Section 2.3 for details):

$$\text{A. For } x(*) < \bar{X}: I(t+1) = I(t) + \left\lfloor \frac{r(t-1)}{x(t-1)} \right\rfloor - \left\lfloor \frac{r(t)}{x(t)} \right\rfloor;$$

$$\text{B. For } x(*) = \bar{X}: I(t+1) = I(t) + \left\lceil \frac{r(t-1)}{x(t-1)} \right\rceil - \left\lceil \frac{r(t)}{x(t)} \right\rceil.$$

3. Get: the value from  $J^*[t+1, I(t+1), x(t)]$  that corresponds to state  $[t, I(t+1), x(t)]$ , using the rounded value of  $I(t+1)$  just computed in (1.2).
4. Compute: one stage reward  $\frac{1}{x(t)}$  for each element of  $\mathbf{X}[x(t)]$ .
5. Compute:  $J[t, I(t), x(t-1)]$ , for each element of  $\mathbf{X}[x(t)]$ , as the sum of (1.3) and (1.4).
6. Choose: from each of the  $J[t, I(t), x(t-1)]$  values computed for  $\mathbf{X}[x(t)]$ , the *maximum* objective function value  $J^*[t, I(t), x(t-1)]$ , and record the corresponding optimal company strength decision

$x^*(t)$ . Ties are broken by selecting the  $J^*[t, *, *]$  that corresponds to the smallest company strength decision  $x^*(t)$ .

7. Enter:  $J^*[t, I(t), x(t-1)]$  and  $x^*(t)$  at the  $[t, I(t), x(t-1)]$  position of  $\mathbf{J}^*[t, I(t), x(t-1)]$  and  $\mathbf{X}^*[x(t)]$  arrays, respectively. Then,
8. Return: to (1.1).

STEP 2. *Determine Optimal Company Strength Policy.*

Given: an initial starting point of  $[1, I(1), x(0)]$ .

Begin: in Stage 1 and work FORWARD in time to Stage 48.

Do: Tasks (2.1) through (2.5) for each of the 48 stages.

1. Look Up: the corresponding value for  $x^*(t)$  from the optimal decision array  $\mathbf{X}^*[x(t)]$  for the current state  $[t, I(t), x(t-1)]$ .
2. Compute: 
$$I(t+1) = I(t) + \frac{r(t-1)}{x(t-1)} - \frac{r(t)}{x^*(t)}$$
 for  $[t, I(t), x(t-1)]$ ,  $x^*(t)$ ,  $r(t)$  and  $r(t-1)$ .
3. Record: the value of  $x^*(t)$  for week  $t$  in  $\pi^* = \{x_1^*, x_2^*, \dots, x_{48}^*\}$ .
4. Replace: the current values of  $[t, I(t), x(t-1)]$  by  $I(t+1)$  and  $x^*(t)$  just determined. Then,
5. Return: to (2.1).

The DP algorithm has been implemented in the same spreadsheet environment used for the SPH and MPH procedures, *LOTUS 1-2-3 for Windows Release 4*. This enables processing times for the two methods to be compared using the same type of

computer and software environment. Computer code for spreadsheet implementation of the DP algorithm is given in Appendix G, along with numerical examples of the  $J^*[t, I(t), x(t-1)]$  and  $X^*[x(t)]$  arrays.

#### *DP Processing Time*

The processing time for one stage using the dynamic programming algorithm took approximately 43 minutes, bringing the total running time required to obtain a solution for 48 periods to approximately 34 hours on a 486/D66 desktop computer. However, DP processing time can be substantially improved by implementing the DP algorithm on a more powerful computer. Even so, it is unlikely that the DP procedure can be successfully used for quick-turn-around analyses required to support "what-if" scenarios that often arise during decision analysis. However, the DP technique has a major advantage over the SPH-MPH procedure in that once the DP results are obtained, the "value" for every feasible company strength policy (e.g., sequence of company strength decisions) is known. In addition, the policies themselves are very easily obtained from the DP solution as a simple table "look up" exercise. Also, the system could "interpolate" for scenarios not in the database.

If DP results for a sufficient number of scheduling scenarios can be pre-computed, then it may be possible to incorporate the results in an "expert system" to support training resource scheduling at the training installation level, or across training installations as well. An expert system can make scheduling results, for whatever scenarios are in the data base, "instantly" available to system users on virtually any 386 or 486 desktop computer.

### *DP Versus Heuristic Results*

Twelve representative examples (of the 441 DP solutions generated) for one-period time lag problems have been selected to compare the DP and heuristic scheduling methods. Table 7 gives the objective function values obtained using DP and the SPH-MPH procedures, and the effectiveness of the heuristic solutions compared to the DP solutions. For initial conditions  $I(t) = 10$  through  $I(t) = 20$ , the differences between the DP and SPH-MPH solutions are negligible (see Appendix G).

From the results, we notice that the SPH-MPH method is more sensitive to initial conditions than the DP procedure. This is indicated by the variability of the objective function values for different initial company strengths (150, 200, 250), for a given initial idle company value. Our second observation is that the objective function values increase as the number of initial idle companies increase (as expected). Averaging the values suggests that the SPH-MPH method (potentially) determines solutions that are 91% (approximately) of the optimal for the one-period lag problem.

Test Case	$I(t)$	$x(t-1)$	SPH-MPH	DP	% Soln
1	0	150	.3047	.3121	98 %
2	0	200	.2831	.3103	91 %
3	0	250	.2652	.3027	88 %
4	1	150	.3071	.3121	98 %
5	1	200	.2981	.3113	93 %
6	1	250	.2721	.3063	89 %
7	2	150	.2091	.3121	98 %
8	2	200	.2942	.3119	94 %
9	2	250	.2773	.3084	90 %
10	3	150	.3108	.3121	99 %
11	3	200	.2983	.3121	96 %
12	3	250	.2831	.3103	91 %

Table 7. Scheduling Results for the One-Period Lag Problem

Figure 18 displays the "quality" objective function values by solution method to allow a visual comparison of results.

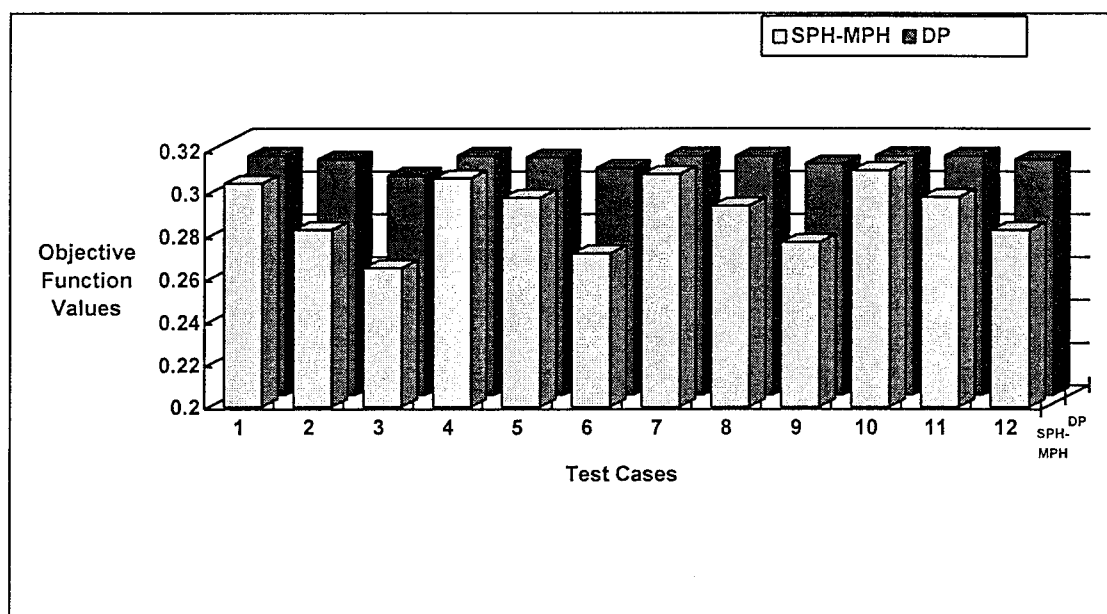


Figure 18. Comparison of Heuristic versus DP Solutions to the 1-Period Lag Problem

Although objective function values for both methods tend to decrease with increases in company strengths, the drop in performance of the SPH-MPH method is much more dramatic when compared to results from the DP procedure for a fixed value of  $I(t)$ .

Next, the DP and SPH-MPH solution methods are compared on the basis of company strength policies (see Figures 19 and 20) and idle training companies (see Figures 21 and 22). These two examples are representative (for the twelve cases considered) of "good" and "poor" heuristic solutions based on solutions that are 98% and 88% percent effective, respectively (see Table 7). The comparison is based on the same initial condition for idle companies,  $I(1) = 0$ , but different initial company strengths,  $x^1(0) = 150$  and  $x^2(0) = 250$ , where superscripts 1 and 2 denote the two cases we are comparing. The optimal objective function values are  $J^{1*}[1, 1, 150] = 0.3121$  and

$J^{2*}[1, 1, 250] = 0.3027$ , and the corresponding heuristic objective function values are 0.3047 and 0.2652, respectively.

Figure 19 compares DP and SPH-MPH company strength decisions for  $x^1(0) = 150$ , and Figure 20 makes the same comparison for  $x^2(0) = 250$ . From Figures 19 and 20, the impact of the initial company strength values of 150 versus 250 is much more evident; especially for the heuristic method.

### *Conclusions*

From the tests and test conditions described for the real-world problem scenarios of Section 6.1, we estimate that the SPH-MPH can determine solutions that are 87% (approximately) of the *utopian* bound for the objective function. The fact that these results were obtained in reasonable time on a 486 microcomputer establishes the potential value of the heuristic methods to basic training management and related decision processes.

For the one-period time lag problem, the heuristic methods achieved results that were 91% of optimal for a small but (seemingly) representative set of test cases. Based on comparing results of different initial company strengths for the one-period time lag problem, it is expected that the performance of the heuristic procedure will likely decline relative to DP as problem size increases.

*Finally, although generating the DP solution is computationally intensive, the DP results are very valuable. Not only because they are of superior quality to the heuristic results, but also because the DP algorithm generates (1) the optimal objective function value, and (2) the decisions to take in each period, for every initial state. Once obtained, albeit at a (potentially) very high cost, the results are very easily obtainable via a simple table "look up" procedure but impractical for "what-if" analysis.*



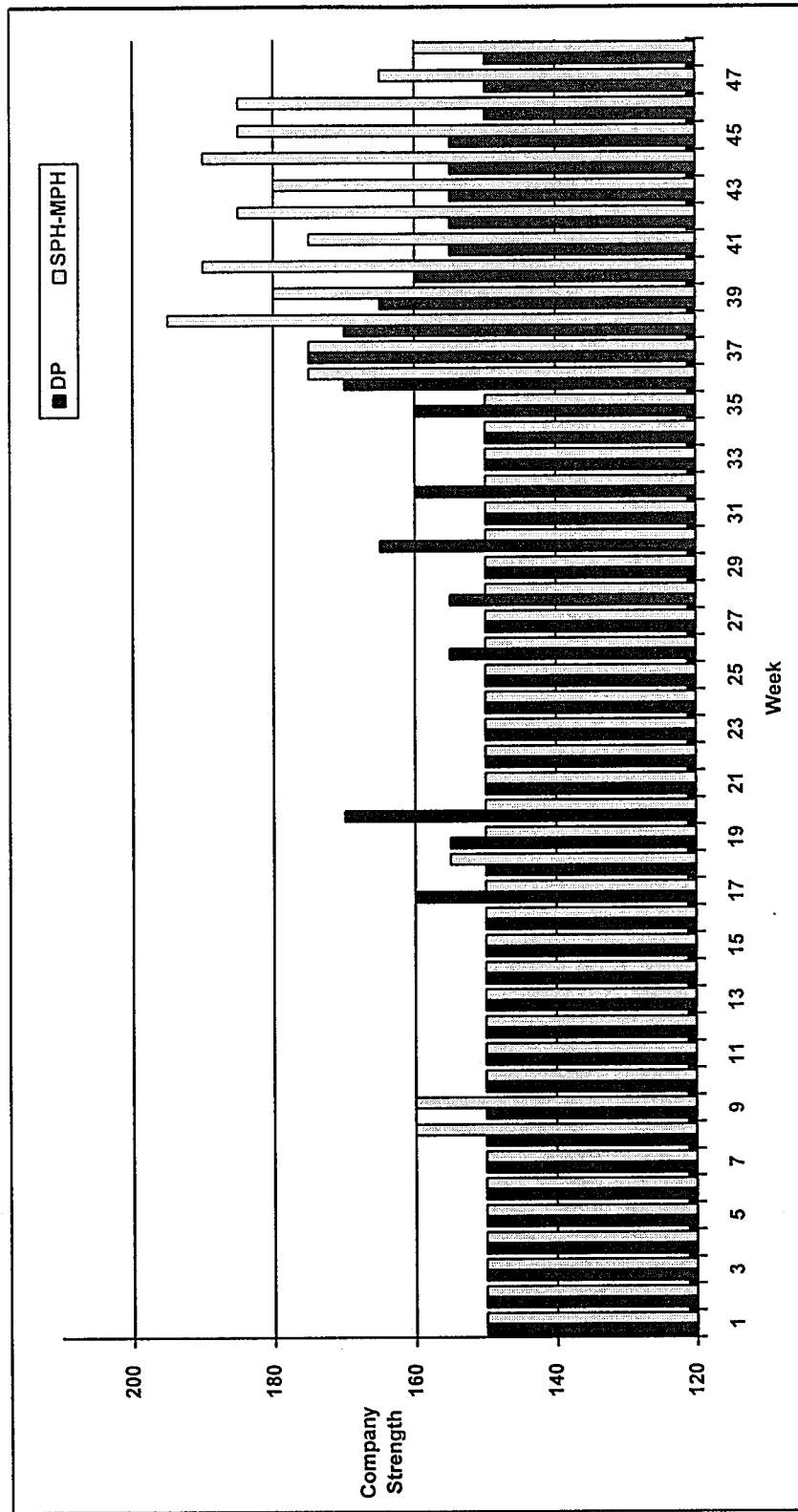


Figure 19. Comparison of Company Strengths for Example 1

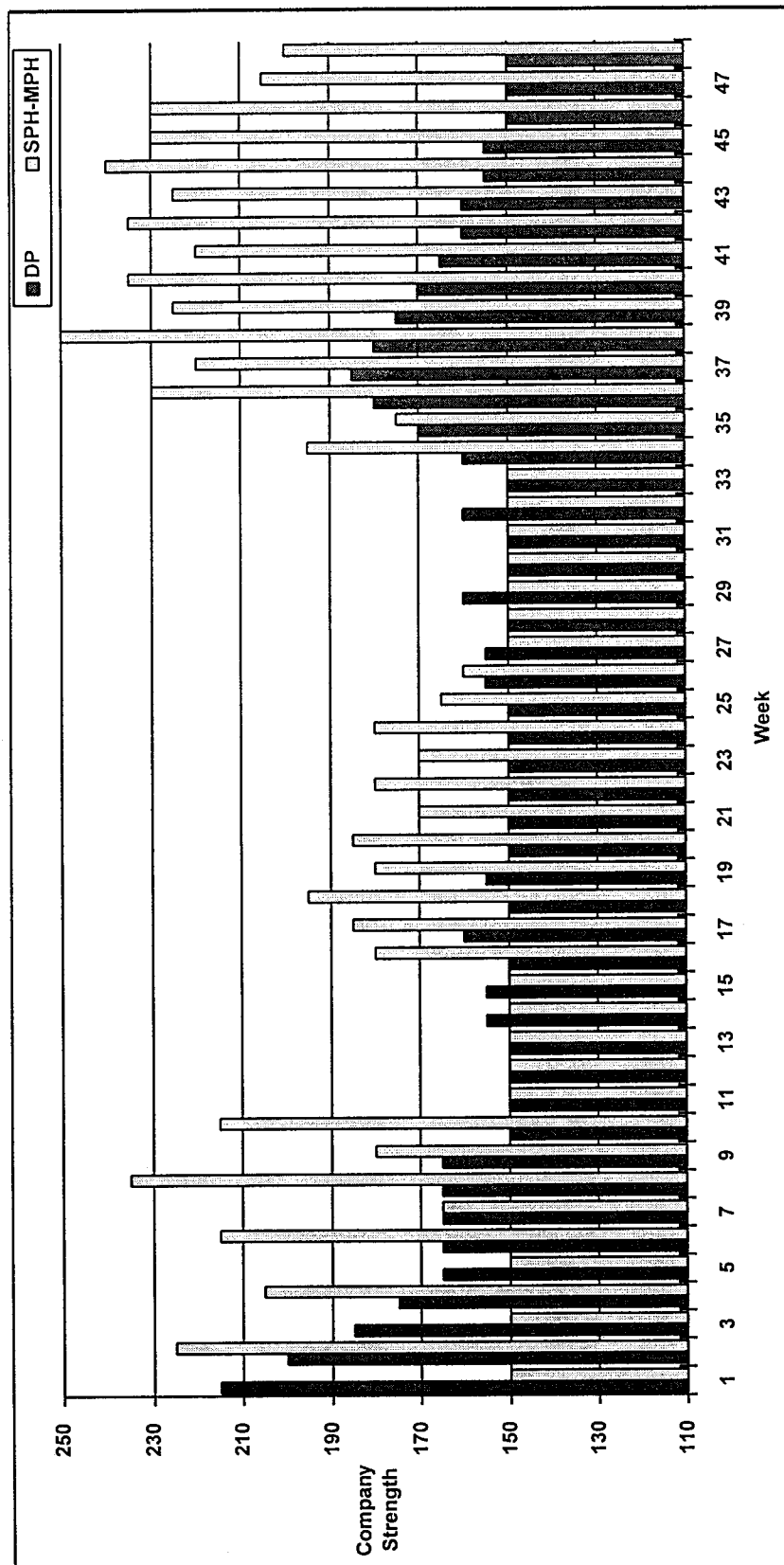


Figure 20. Comparison of Company Strengths for Example 3



Figure 21. Comparison of Idle Training Companies for Example 1

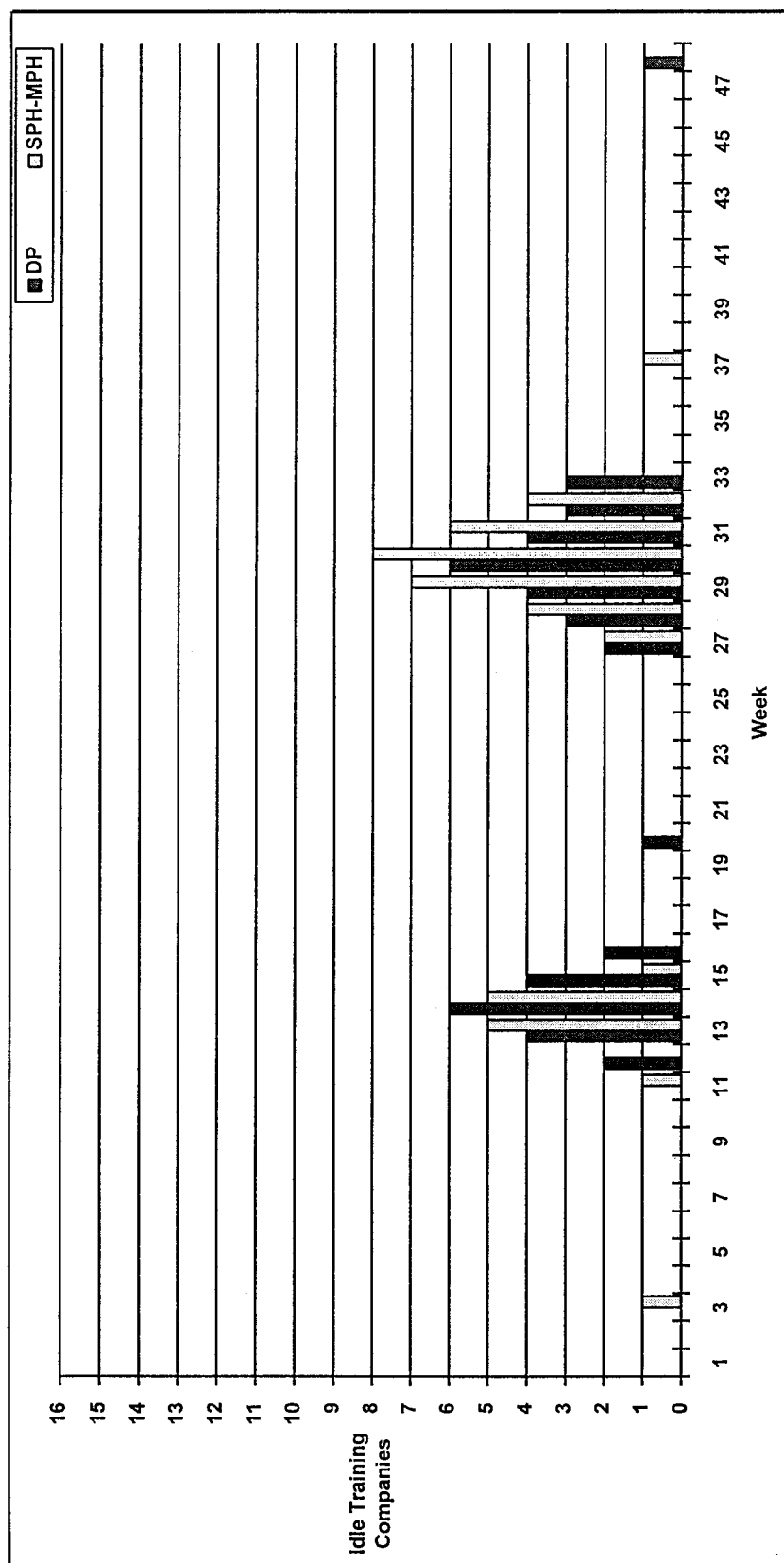


Figure 22. Comparison of Idle Training Companies for Example 3

## 7. CONCLUSIONS

This dissertation solves a complex scheduling problem of practical interest to the United States Army that has apparently not been previously reported in the literature. Specifically, the problem is one of scheduling *reusable* training resources for the Army's Basic Combat Training program over a finite planning horizon of  $T$  stages.

### 7.1 Summary and Contributions of Work

One contribution of this work to military systems engineering is the mathematical formulation of the optimal resource scheduling problem that accounts for important dynamics of the basic training process. Notable dynamic characteristics of the training system model include:

1. seasonal arrival of recruits that represents demand for training resources;
2. varying training company strength; and
3. varying training cycle length.

In the real-world basic training problem, recruit arrival is a random process which makes the demand for training resources stochastic. However, we estimate recruit arrivals ahead of time for each of  $T$  time periods which makes our problem deterministic.

The primary optimization (or suboptimization, as the case may be) objective of the basic training problem is based on a key measure of training system performance; namely, the "quality" of the training program as measured by the instructor-to-student ratio. A second objective (given) is the minimization of training program cost; also a very desirable criterion. Based on the primary objective, two decision elements are modeled that reflect realistic decisions made in practice: training company strength and

training cycle length. The problem is to determine the appropriate strength and cycle length for training companies at the beginning of each of the  $T$  time periods to meet deterministic demand for training resources as measured by recruit arrivals.

A second major contribution of this work is the implementation of scheduling methods for solving the training resource problem. Two decision models are considered. The first is based on *dynamic programming*; an exact optimization where the scheduling decisions at each stage of the planning horizon are the ones that maximize the sum of the training quality measure for the current stage plus the best value for the objective function (total training quality) that can be achieved for future stages. Computer implementation of the dynamic programming technique has been limited to a simplified version of the real-world basic training problem due to the massive size of the original problem.

The second decision model consists of two heuristics applied in three phases to obtain a suboptimal resource scheduling solution. The first of the two heuristics is a forward single-pass heuristic (SPH) that generates a "good" initial feasible training resource schedule. The second heuristic is a backward multi-pass heuristic (MPH) that sequentially (in time) and iteratively (in decisions) refines the initial feasible schedule until no further improvements can be made. Based on experimental results, the single- and multi-pass heuristics generate solutions that are approximately 87% effective (based on the *utopian* value of the training quality performance measure). This is a significant improvement over the approximately 74% effective solutions achieved with heuristics-used-in-practice for the same set of test problems. The SPH and MPH include a fully automated heuristic policy iteration step that is motivated by the policy iteration approach to solving dynamic programs.

A third major contribution of this dissertation is the development of an operational *Decision Support System for Army Basic Combat Training Resource*

*Management per Year*, or *ARMY*, that fully implements the SPH and MPH heuristics for solving real-world basic training resource scheduling problems. The decision support system has been designed to support three major levels of decision making: strategic planning at the Department of the Army level, training installation management at TRADOC Headquarters, and operational control of the training program at the training installation. Experiments with *ARMY* have demonstrated that the system can be used to estimate

- minimal training structure required to meet current and future annual training requirements;
- training resource utilization; and
- basic training program costs including the overall annual cost of the basic training program, the annual costs of specific program resources, and the training program cost per trainee.

*ARMY* also supports the analyses of a variety of training installation and training program management issues, such as,

- examining the impact of consolidating, closing, or reducing the size of training installations on the Army's ability to meet future training missions;
- evaluating the economic impact of different resource utilization policies;
- evaluating training readiness (as a function of training capacity and training program throughput) by varying training parameters, such as, recruiting levels, training program durations (course lengths), or levels of available training resources; and

- forecasting training resource requirements for the basic training program and estimating basic training program costs.

Potential benefits of the *ARMY* system to the to the Army, and perhaps to the Department of Defense, in general, include:

- improved forecasting of training resource requirements;
- improved training resource scheduling;
- improved resource utilization through tighter control of facilities, equipment, supplies, and manpower; plus
- considerable cost savings associated with each of the improvements to training management cited above.

## 7.2 Suggestions for Future Research

This work has concentrated mainly on the formulation of a model for the Army's Basic Combat Training program, and implementing a decision model for scheduling one major training resource; basic training companies. The dissertation concludes with a list of potential research areas to be studied in the future.

- In the model, permit recruits to be backlogged in the model when recruit arrivals exceed training capacity. This means that demand for training resources (i.e., training companies) can be backlogged, and therefore,  $I_j(t) < 0$  is permitted in the model.
- Make the basic training model more realistic by incorporating (1) recruit failures, (2) recycling of recruits who fail basic training and (3) account for the added cost of recycling.



- Incorporate additional direct and indirect costs into the Resource Costing Module of the *ARMY* system.
- Investigate the effect of incorporating stochastic processes for recruit arrivals, recruit failures, and recruit recycling on demand for training resources and on resource scheduling.
- Extend the model and the decision support system to include other Army training programs (AIT, OSUT, training programs for commissioned and noncommissioned officers, and specialty training programs, such as, Airborne School, Ranger School, etc.), and training programs of other branches of military service (Air Force, Navy, and Marines).
- Improve the efficiency of the computer code for the single- and multi-pass heuristic scheduling algorithms.
- Extend the optimal dynamic programming implementation to larger, more realistic scheduling problems.

In conclusion, the model of the basic training program and the scheduling methodologies presented in this dissertation will hopefully motivate further research efforts in this important area of military operations research.

## Appendix A:

### SUMMARY OF LITERATURE SURVEY

An extensive search of the literature was made for papers related to military training programs, training resource scheduling, training base or training installation management, military base closures or base realignment, and other relevant issues. The primary sources for the literature search were the *Compendex* and *MathSci* databases, and the Defense Technical Information Center (DTIC). *Compendex* and *MathSci* are international databases referencing over 4500 and 3200 journals, conference proceedings, and technical reports, respectively, in such areas as engineering, mathematics, operations research, and computing. DTIC is a central depository of scientific and technical research, and data collection, for the Department of Defense. The *MathSci* and DTIC databases were searched from 1980 through 1993 and *Compendex* from 1986 through 1993 using approximately 100 key words that generated more than 28,000 citations (see Table 8 below). Initial screening, done by title, identified approximately 200 papers thought to have some connection (however remote) to the basic training problem. These 200 were further screened by reading the abstracts. This step reduced the number of papers to approximately eighty, of which fifty (or so) were related to the economic lot-sizing problem. Not one of the remaining thirty papers was directly related to the basic training problem presented here.

Table 8 summarizes the literature search. The value in each cell gives the number of references initially obtained by keyword and year. Totals by key word are given in the right most column and the last row totals references by year.

Key Words/Years	1993	1992	1991	1990	1989	1988	1987	1986	Total
Air Force	8	7	3	9	1	3	9	13	53
Army	74	188	220	172	138	266	229	297	1584
Army Bases						1			1
Army Battalion Training						1			1
Army Facilities		1		1	1				3
Army Installations				1	1		1		3
Army Recruitment								1	1
Army Reserve Pers Training			1						1
Army Training Management								1	1
Decision Support System	156	493	349	145	92	101	146	28	1510
Decision Theory-Mil Appl		1	1		6	8	3	2	21
Decision Theory-Mil Purp					1		6	11	18
DP	28	143	134	89	101	96	157	115	863
DP Algorithm					3	4			7
DSS	11	62	66	50	52	69	84	68	462
DSS Design					1				1
DSS Military Application		9		2					11
DSS Military Purposes		1							1
Dynamic Programming	63	17	19	18	11	16		19	163
Manpower	27	93	92	98	105	188	187	232	1022
Manpower, Personnel, Tng				1					1
Manpower Allocation					1			1	2
Manpower Analysis	7			1					8
Manpower Assessment					1				1
Manpower Costs		1						1	2
Manpower Forecasting								2	2
Manpower Issues							2	1	3
Manpower Management						1		1	2
Manpower Modeling					1				1
Manpower Opt				1					1
Manpower Planning		7	4	1	1	1		9	23
Manpower Planning Model			3	1	1				5
Manpower Policies			1						1
Manpower Resources			2	1					3
Manpower Resources Mgt				1					1
Manpower Scheduling				1		1			2
Manpower System Costs			1						1
Manpower Training		1					1		2
Military Engr-Math Models			3		2				5
Military Engr-Personnel		6	4	2	9	1	2	11	35
Military Engr-Personnel Tng			1	1		7			9

Military Applications	38	246	249	179	291	328	301	306	1938
Military Base Closures			1						1
Military Bases	2								2
Military Installations			3			1			4
Military Personnel	1	1							2
Military Training						1			1
Navy	61	160	172	136	138	186	258	289	1400
Recruit	2	10	3	3	41	30	3	8	100
Recruiting	4		21	8	8	7	9	9	66
Recruitment	3	35	43	43	33	26	37	31	251
Resource Allocation			40		35	45			120
Resource Allocation Algo			1	2	1			1	5
Resource Allocation Pblm	15	3	2	1			2	38	61
Resource Assignment Pblm		2							2
Resource Balancing Pblm					1				1
Resource Management			22		56	12	23	24	137
Resource Management Model	6			13	1	1	1		22
Resource Modeling	1				1		1		3
Resource Planning			3	1	1	6	7	3	21
Resource Scheduling		1	3	5		1			10
Resource Scheduling Costs						1			1
Scheduling	342	983	1193	938	816	1059	1174	1030	7535
Scheduling Algorithms	1	19	4		11	10	2	12	59
Scheduling Analysis					3				3
Scheduling Applications		46	64		3	1			114
Scheduling Automation		2					3		5
Scheduling Calculations		1			1				2
Scheduling Computer Applic		33	4		29	33	35		134
Scheduling Computer Simul		2	5	4	4	11	3	5	34
Scheduling Costs		1	1			1	1		4
Scheduling Heuristics		1						1	2
Scheduling Math Models		18	12		43	75			148
Scheduling Management			1						1
Scheduling Problems						2		1	3
Scheduling Personnel						1			1
Scheduling Programs						1			1
Scheduling-Military Purposes				1		1			2
Training	422	1657	1526	1258	902	1325	1452	1585	10127
Training Activities Schedule						1			1
Training Costs								1	1
Training Facilities		1				1	1		3
Training Installations					1				1

Training Management		1	1	1					3
Training Missions						1			1
Training Process				1					1
Training Program		5	6	5			4	18	38
Training Resources		1			1				2
Training Scheduling Algo						3			3
Training Strategies		1				1	1		3
Training Systems		3		1			1		5
Training System Analysis				1					1
Training System Design				1					1
US Air Force	2	27	19	13	5	2	2	11	81
US Air Force Pers		1							1
US Army	2	14	5		2	5	1	12	41
US Army Installations					1				1
US Navy	7	12	15	4	11	1	5	3	58
<b>TOTAL</b>	1283	4317	4323	3215	2969	3945	4154	4201	28406

Table 8. Literature Search Results

## Appendix B:

## TRAINING BASE SCENARIO DECISION PROCESS

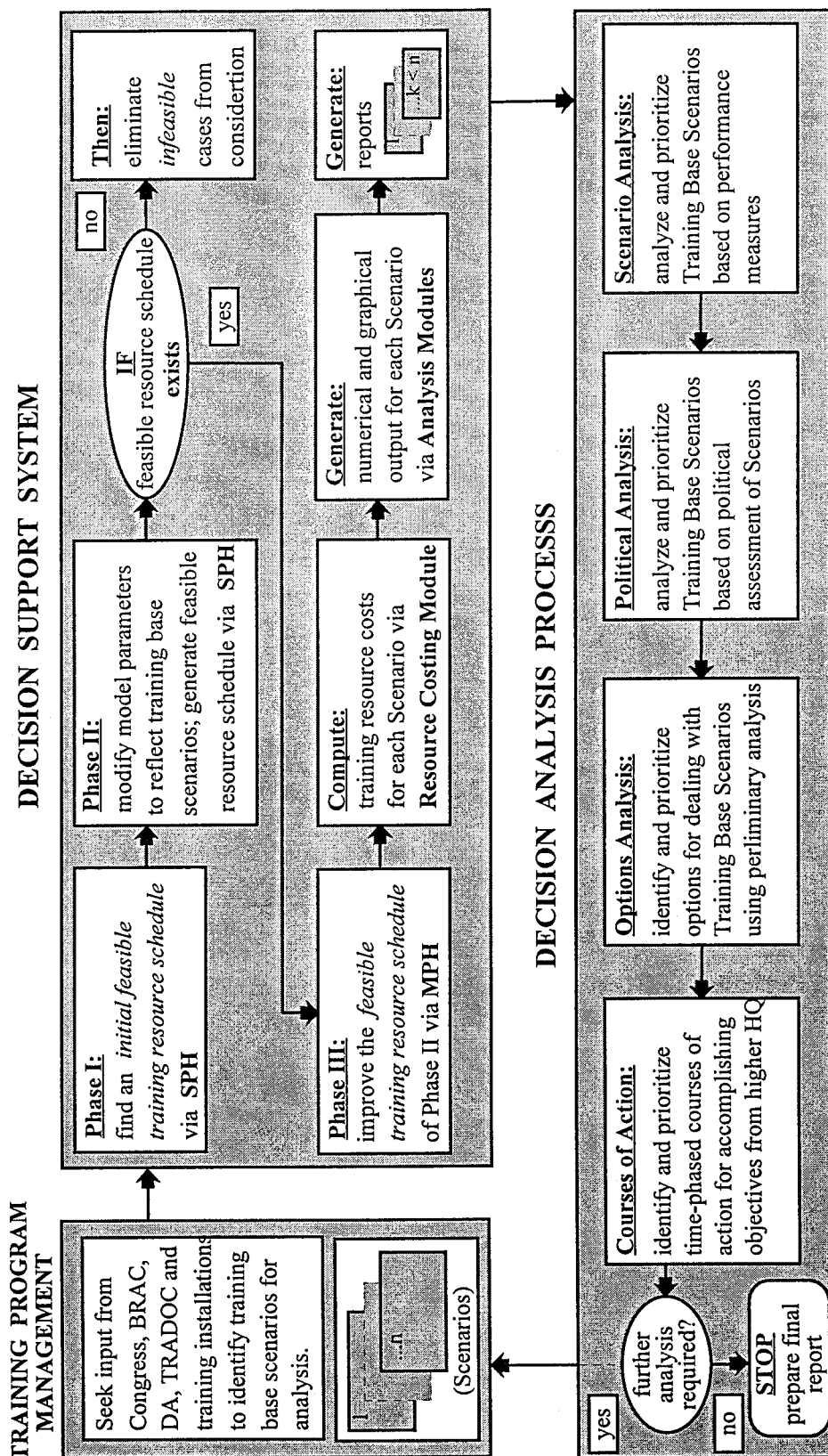


Figure 23. Training Base Scenario Decision Process

## Appendix C:

### ILLUSTRATIVE SESSION WITH THE DECISION SUPPORT SYSTEM

Training Base Scenario 11 is used to illustrate numerical and graphical output from the *ARMY* decision support system (DSS). Scheduling results are based on 1988 training installation structure and (notional) projected recruiting objectives for 1989 and 1990. Initially, there are 130 training companies available at the beginning of 1989. Historical data is used to "warm-up" the model so that idle companies in week 1 of 1989 reflects a realistic initial starting point. The training company balance equation given by equation (12) of Section 2.3 determines the number of idle training companies in each week. *Company Strength(s)* and training *Cycle Length(s)* (Figure 24 below) are initialized at their lower and upper bounds of 150 recruits and 10 weeks, respectively. Training company deactivation is applied at the beginning of each year of the planning horizon (according to Scenario 11 (fifteen companies are deactivated at the beginning of both 1989 and 1990) . This is reflected in the number of training companies available at the beginning of 1989 and 1990, 115 and 100 companies, respectively (see *BCT Co*, Figure 24). The *Less Deactivated Co* row is used when training company deactivation is applied across the planning horizon. The *Co from [10, 9, 8] Wk Cycle* rows serve as the work space for the Resource Scheduling Algorithm. Some summary statistics are given at the end of the planning horizon in Figure 24. Figures 25 through 29 graph idle training companies at progressive stages of the heuristic scheduling procedure.







	Year																			1990	
	Recruits																			128000	
	BCT Co																			100	
1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1989	1990	1990
33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50				
16	12	8	9	9	7	4	2	1	0	0	3	8	9	10	12	0	0				
2504	2765	3044	3315	3564	3562	3560	3558	3557	3604	3662	3696	3745	3535	3301	3066	2671	2644				
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1				
2254	2488	2739	2984	3208	3205	3204	3202	3201	3244	3296	3327	3370	3181	2971	2759	2404	2380				
150	150	150	210	240	240	240	240	240	245	215	230	250	240	240	240	230	180				
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10	10				
15	17	18	14	13	13	13	13	13	13	15	14	13	13	12	11	10	13				
42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59				
13	14	15	13	11	10	11	12	12	15	17	18	14	13	13	13	13	13				
0.0067	0.0067	0.0067	0.0048	0.0042	0.0042	0.0042	0.0042	0.0042	0.0041	0.0047	0.0043	0.004	0.0042	0.0042	0.0042	0.0043	0.0056				

Figure 24. Continued





														Summary	
		1990	1990	1990	1990	1990	1990	1990	1990	1990	1990	1990	1990	1989	1989-90
87	88	2	1	0	0	0	0	0	0	0	0	0	0	5.54	2.88
4	3349	3347	3392	3447	3479	3524	3327	3107	2885					136000	128000
0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.10	0.10
3016	3014	3013	3053	3102	3131	3172	2994	2796	2597					122398	115198
245	245	245	245	215	190	235	240	225	210					186.35	190.63
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10.00	10.00
12	12	12	12	14	16	13	12	12	12	12	12	12	12	13.63	12.42
96	97	98	99	100	101	102	103	104	105					654	596
10	11	11	14	16	14	11	12	12	12						
														SPH MPH	
0.0041	0.0041	0.0041	0.0041	0.0047	0.0053	0.0043	0.0042	0.0044	0.0048					0.527	0.529

Figure 24. Continued

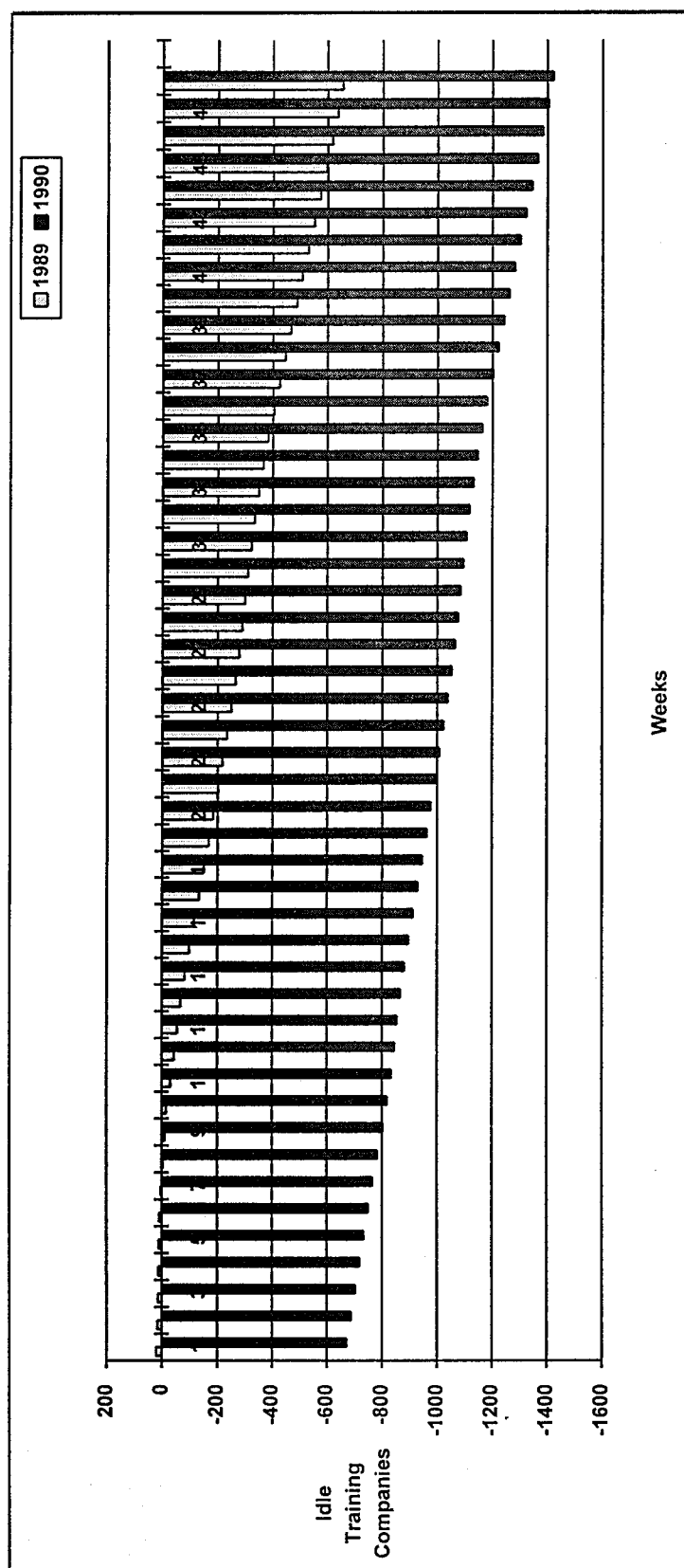


Figure 25. Training Company Shortfalls Before Beginning Heuristic Scheduling

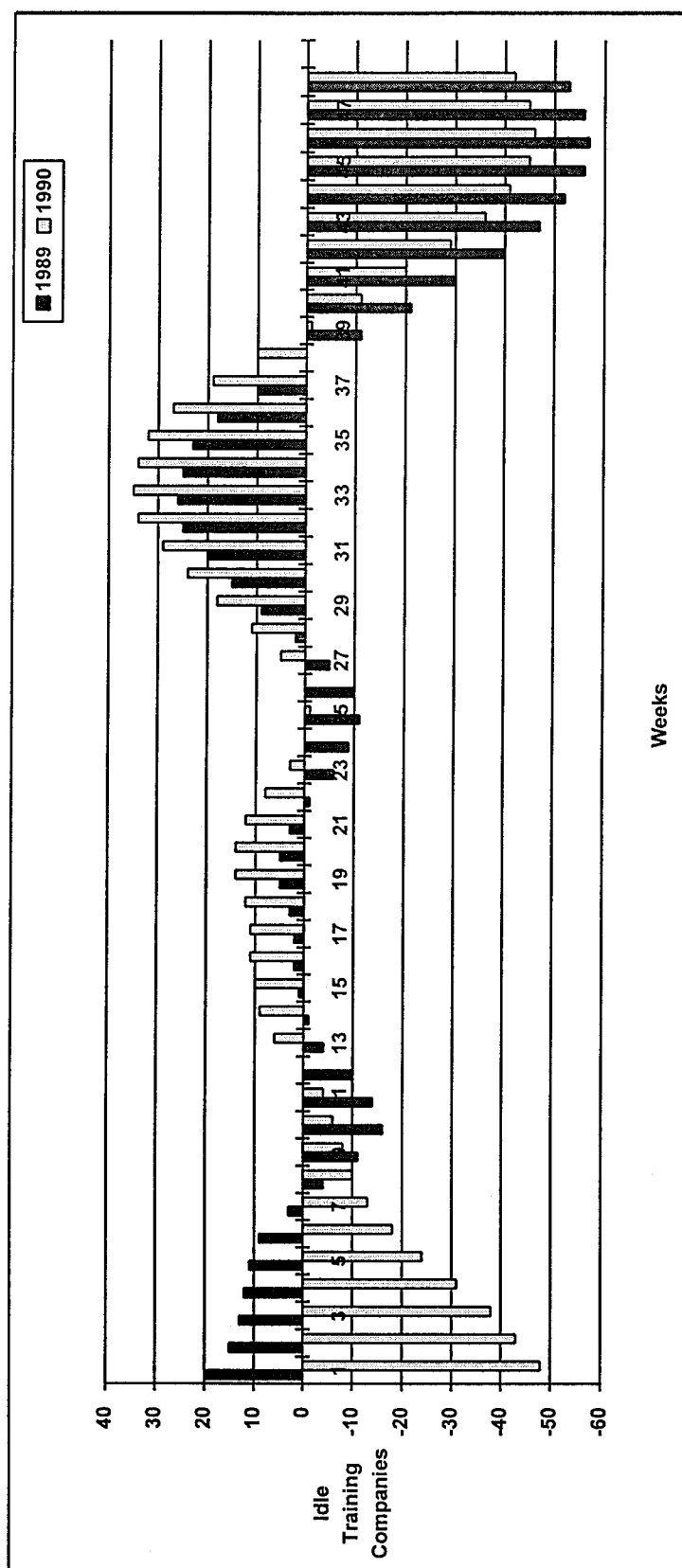


Figure 26. Training Company Shortfalls After the Resource Scheduling Algorithm of Phase I

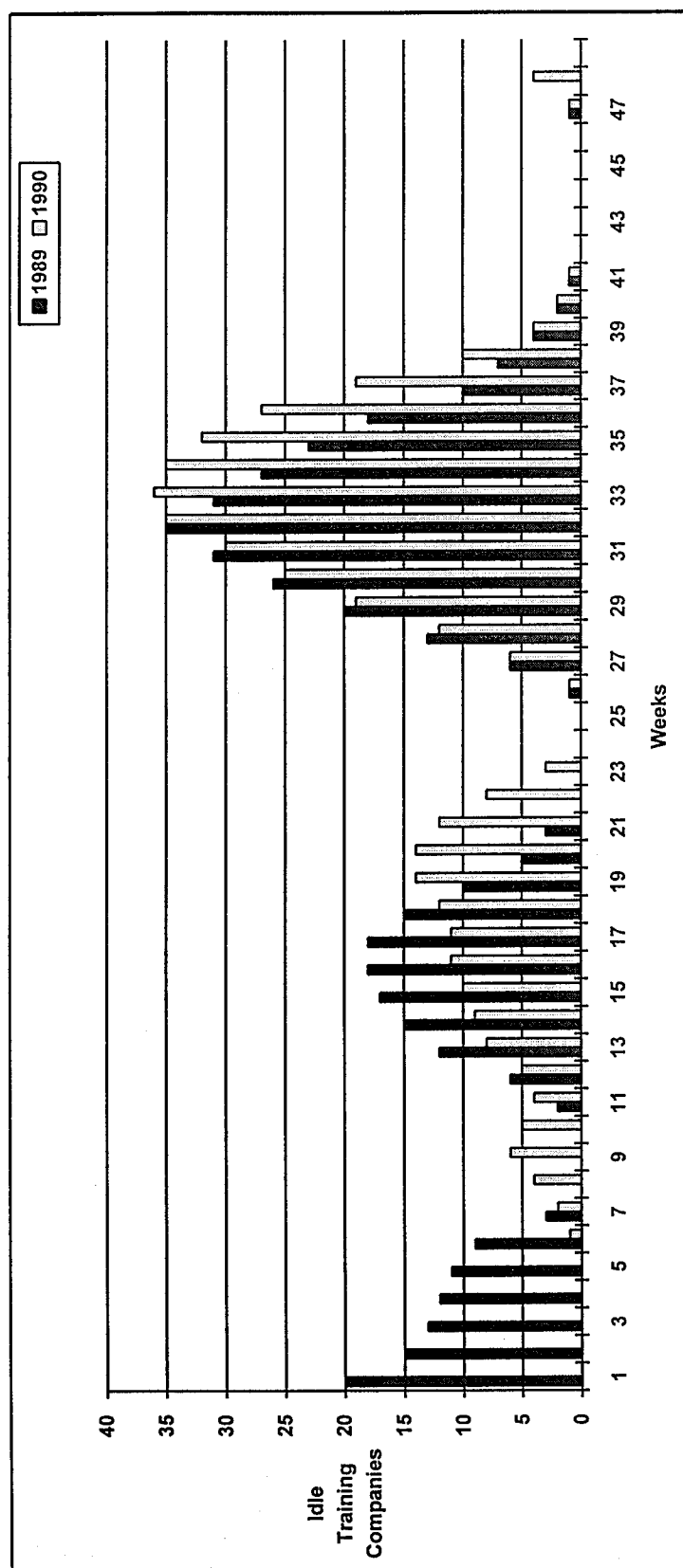


Figure 27. Results Showing the Initial Feasible Training Resource Schedule of Phase I



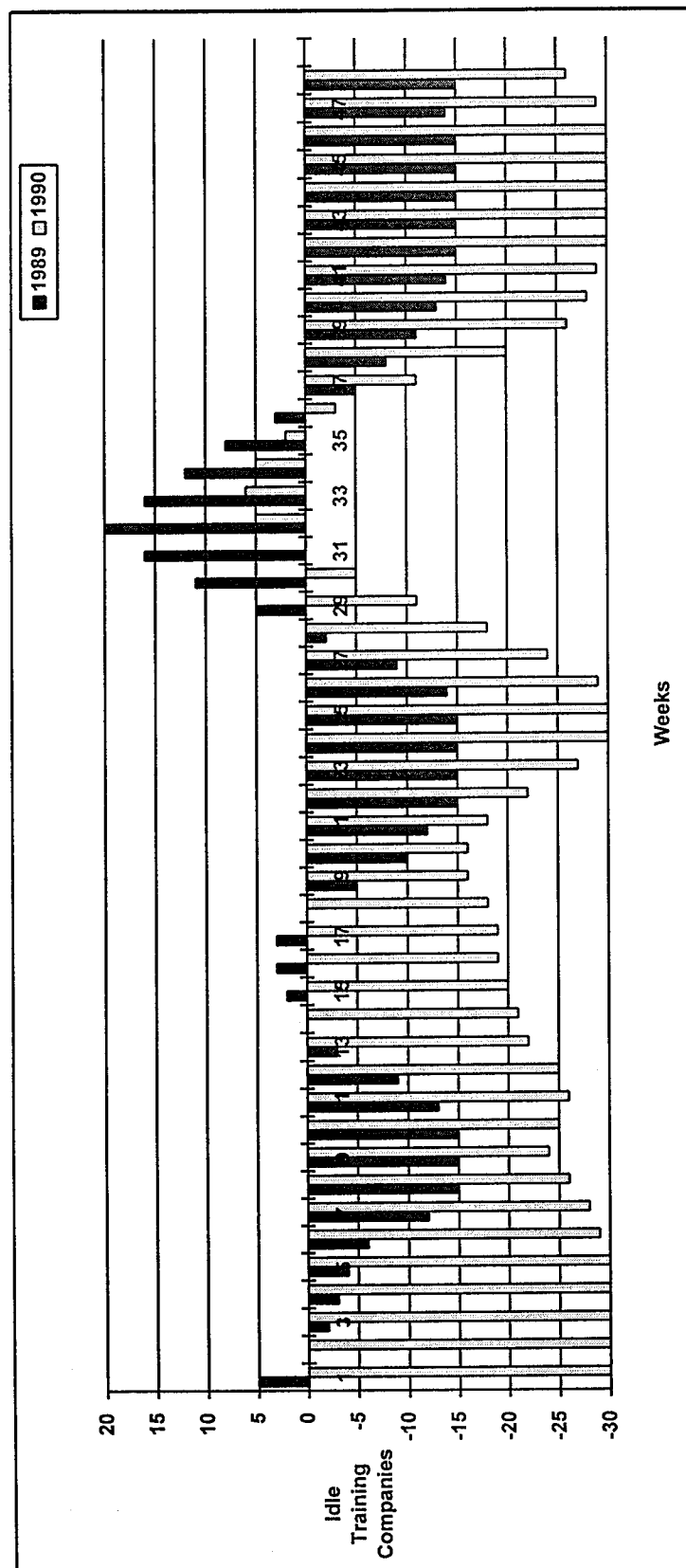


Figure 28. Training Company Shortfalls After Deactivating Training Companies in Phase II

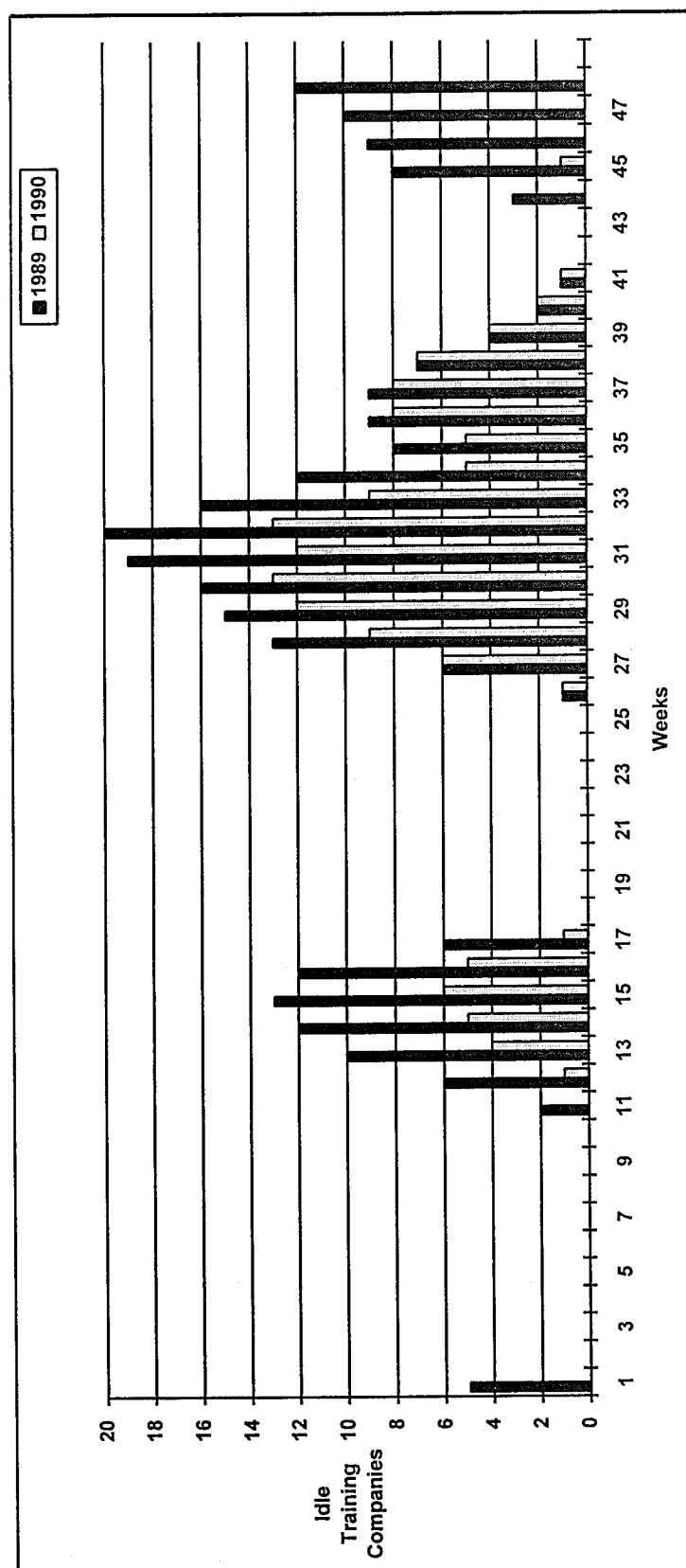


Figure 29. Idle Training Companies After Company Strength Policy Improvement Algorithm via Multi-Pass Hueristic

**Appendix D:****INDIRECT TRAINING PROGRAM COSTS<sup>1</sup>****BASE OPERATIONS SUPPORT**BASE SUPPORT SERVICES

Administration	Other Base Services
Retail Supply Operations	Morale, Welfare and Recreation
Maintenance of Installation Equipment	Unaccompanied Housing
Other Personnel Support	

FACILITY SUPPORT SERVICES

Utilities	Other Engineer Support
Maintenance and Repair of Real Property	Environmental Compliance
Minor Construction	

**BASE SUPPORT SERVICES**ADMINISTRATION

ADP Facilities and Equipment	Headquarters Administration & Command
ADP Services	Inspector General
Analysis & Resources Management	Internal Review & Audit Compliance
Base Audiovisual Activities	Legal Services
Base Telephone & Telecommunications	Local Automation
Chaplain Activities	Manpower Management
Command Element	Printing & Reproduction
Comptroller and Accounting & Finance	Program & Budget Activities
Correspondence Control	Public Affairs
Dependent Schools	Records Management
Equal Employment Opportunity	Safety

RETAIL SUPPLY OPERATIONS

Clothing Issue Points	Personnel & Community Affairs
Fuel Management	Ration Distribution
Military Clothing & Equipment Sales	Reenlistment Activities
Purchasing & Contracting	Social Activities
Military Personnel Management	Training
Other Dining Costs	Treatment Programs
Other Personnel Costs	

---

<sup>1</sup>Courtesy of the Directorate of Resource Management (DRM), Fort Huachuca, Arizona.

OTHER BASE OPERATIONS

Base Transportation  
 Vehicle Operations & Maintenance  
 Civil Disturbance Activities  
 Corrections & Confinement  
 Counterintelligence Operations  
 Disaster Preparedness  
 Household Goods  
 Installation Traffic Management  
 Laundry & Dry Cleaning  
 Leased Physical Security Vehicles  
 Liaison & Apprehension  
 Military Police Activities  
 Museums  
 Nuclear & Chemical Activities

Other Base Support  
 Physical Security Activities  
 Plans, Training & Mobilization Support  
 Rail Services  
 Reserve Component Support  
 Security Administration Support  
 Security Guards & Police  
 Security Operations  
 Small Security Equipment Purchases  
 Small Security Equipment Installation  
 Traffic Control  
 Training Devices  
 Training Facilities  
 Water Port Services

MORALE, WELFARE & RECREATION

Auto, Craft & Hobby Shops  
 Bowling Alley  
 Child Care Centers & Activities  
 Clubs, Messes & Restaurants  
 Theaters, Marinas, Golf Courses & Pools  
 Other Non-appropriated Resale Activities  
 Community Support Activities  
 Entertainment Tickets & Tours  
 Other Special Services  
 Family Member Employment Program  
 Family Support Programs

Financial Planning-Consumer Affairs  
 Foster Care  
 Libraries  
 Recreation Activities & Centers  
 Relocation Services  
 Selected Medical & Dental Clinics  
 Social Activities & Support Groups  
 Sports Programs  
 Welfare Funds  
 Youth Activities  
 Youth Development Activities

UNACCOMPANIED HOUSING

Moving Unaccompanied Personnel  
 Procurement, Control, Issue, Repair &  
 Replacement of Unaccompanied Housing  
 Furnishings

Unaccompanied Housing Activities  
 Operation of Unaccompanied Housing

## **FACILITY SUPPORT SERVICES**

### **UTILITIES**

Air Conditioning & Refrigeration Plants  
Electric Service & Systems  
Other Utilities  
Sewer Service & Systems

Steam & Hot Water Heating Plants  
Utilities at Inactive Installations  
Water Service Systems

### **MAINTENANCE & REPAIRS OF REAL PROPERTY (MRP)**

Buildings  
Facilities Engineering Shops  
Grounds  
Miscellaneous Maintenance  
MRP of Inactive Installations

Railroads  
Surfaced Areas  
Tool Issue  
Utility Systems  
Suspense Accounts (Engr Shops & Tools)

### **MAINTENANCE OF INSTALLATION EQUIPMENT**

Audiovisual Equipment  
Chaplain Equipment  
Band Equipment  
Electronics Equipment  
Engineering Equipment  
Communications Equipment  
ADP Equipment  
Food Service Equipment  
Training Equipment

General Equipment  
Non-Tactical Equipment & Vehicles  
Other Commodities  
Other Support Equipment  
Personnel Support Equipment  
Rail Equipment  
Airlift Support Equipment  
Unaccompanied Personnel Furnishings

### **OTHER ENGINEERING SUPPORT**

Custodial Services  
Engr. Public Works and Management  
Engineering Support (Inactive Install)  
Equipment

Self-Service Centers  
Stock & Supply Operations  
Stock Distribution  
Storage & Warehousing

### **MINOR CONSTRUCTION**

Alterations & Construction (Active Install)

Alterations & Construction (Inactive Install)

### **OTHER PERSONNEL SUPPORT**

Bands  
Civilian Personnel Management  
Contractor Food Operations  
Dining Facilities

Education  
Food Service  
Information Activities

## Appendix E:

### COST FACTORS

Resource cost data from the *1993 Fort Benning Study*, discussed in Chapter 5, is used to compute fixed and variable training resource cost factors. Results are given below. Chapter 6 illustrates how the cost factors may be applied to estimate training resource costs for training resource schedules obtained via the heuristic scheduling approaches of Chapter 4.

Fixed costs are generally assessed at the beginning of each year (YR) of the planning horizon based on the number of training battalions, or the number of training companies, available at the beginning of each year. Fixed and variable costs are computed by training battalion (BN) and training company (CO). Variable costs are per recruit are also computed. Costs per start for battalions and training companies reflects the cost per training cycle for those units, where a battalion cost is assessed for each set of five training companies that start training in a given week (see Section 6.1 for further details). Cost factors are summarized for each cost item at the end of Appendix E.

**PERSONNEL IN-PROCESSING**

Cost Factors and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Civilian employee for soldier in-processing:	\$27,000				
General Supplies: \$3,000/CO. Assuming 200 recruits per company, $\frac{\$3,000/\text{CO}}{200 \text{ Recruits}/\text{CO}} = \$15/\text{Recruit}$					\$15
Organizational Clothing: \$14,000/CO. Assuming 200 recruits per company, $\frac{\$14,000/\text{CO}}{200 \text{ Recruits}/\text{CO}} = \$70/\text{Recruit}$					\$70
<b>Subtotal</b>	\$27,000				\$85

**BASIC TRAINING SUPPORT**

Cost Factors and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Employees for administrative support:	\$83,000				
Soldier Item Issue: \$23,000/CO. Assuming 200 recruits per company, $\frac{\$23,000/\text{CO}}{200 \text{ Recruits/CO}} = \$115/\text{Recruit}$					\$115
Company Item Issue: \$2,000/CO. Assuming 6 starts per year, $\frac{\$2,000/\text{CO}}{6 \text{ Starts/YR}} = \$333/\text{CO/Start}$				\$333	
General Supplies Issue: \$6,000/CO. Assuming 6 starts per year, $\frac{\$6,000/\text{CO}}{6 \text{ Starts/YR}} = \$1000/\text{CO/Start}$				\$1000	
Weapons Cleaning Kits: \$6,000/CO. Assuming 200 recruits per company, $\frac{\$6,000/\text{CO}}{200 \text{ Recruits/CO}} = \$30/\text{Recruit}$					\$30
Load Bearing Equipment (LBE): \$40,000/CO. Assuming 200 recruits per company, $\frac{\$40,000/\text{CO}}{200 \text{ Recruits/CO}} = \$200/\text{Recruit}$					\$200



Medical Supplies Issue: \$5,000/CO. Assuming 6 starts per year, $\frac{\$5,000/\text{CO}/\text{YR}}{6 \text{ Starts}/\text{YR}} = \$833/\text{CO}/\text{Start}$				\$833	
<b>Subtotal</b>	\$83,000			\$2166	\$85

### SUPPLY OPERATIONS

Cost Factors and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Sewing Machine Operator:			\$19,000		
Tailor:			\$20,000		
Clerk to transcribe data and process records:			\$19,000		
<b>Subtotal</b>			\$61,000		

### MAINTENANCE OF MATERIALS: Direct Support (DS) & General Support (GS)

Cost Factors and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Worker for Organizational Maintenance:			\$17,000		
Workers for DS/GS Maintenance:			\$51,000		
Weapons Maintenance: \$14,000/CO. Assuming 200 recruits per company, $\frac{\$14,000/\text{CO}}{200 \text{ Recruits}/\text{CO}} = \$70/\text{Recruit}$					\$70

Maintenance of Load-Bearing Equipment: \$2,000/CO. Assuming 200 recruits per company, $\frac{\$2,000/\text{CO}}{200 \text{ Recruits}/\text{CO}} = \$10/\text{Recruit}$					\$10
DS/GS Repair Parts: \$42,000/CO. Assuming 200 recruits per company, $\frac{\$42,000/\text{CO}}{200 \text{ Recruits}/\text{CO}} = \$210/\text{Recruit}$					\$210
<b>Subtotal</b>			\$68,000		\$290

### TRANSPORTATION SERVICES

Cost Factors and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Training Company Driver Costs:			\$56,000		
Administrative Vehicle Costs: \$15,000/BN = \$15,000/5 CO. Assuming 6 starts per year, $\frac{\$15,000/\text{BN}/\text{YR}}{6 \text{ Starts}/\text{YR}} = \$2500/\text{BN}/\text{Start}$		\$2500			
Training Company Vehicle Costs: \$11,000/CO. Assuming 6 starts per year, $\frac{\$11,000/\text{CO}/\text{YR}}{6 \text{ Starts}/\text{YR}} = \$1833/\text{CO}/\text{Start}$				\$1833	
<b>Subtotal</b>		\$2500	\$56,000	\$1833	

**LAUNDRY SERVICES**

Cost Factor and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Laundry Services:					\$145
<p>\$29,000/CO.</p> <p>Assuming 200 recruits per company,</p> $\frac{\$29,000/\text{CO}}{200 \text{ Recruits}/\text{CO}} = \$145/\text{Recruit}$					
<b>Subtotal</b>					<b>\$145</b>

**FOOD SERVICES**

Cost Factor and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Food Services:					\$778
<p>\$933,000/CO.</p> <p>Assuming 6 starts per year and 200 recruits per company,</p> $\frac{\$933,000/\text{CO}/\text{YR}}{(6 \text{ Starts}/\text{YR}) (200 \text{ Recruits}/\text{CO})} = \$778/\text{Recruit}$					
<b>Subtotal</b>					<b>\$778</b>

**PERSONNEL SUPPORT**

Cost Factor and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Clerk for Personnel Support:			\$26,000		
<b>Subtotal</b>			<b>\$26,000</b>		

## AMMUNITION

Cost Factor and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Ammunition Issued to <u>Recruits</u> for Qualification on Individual Weapon (M16): ball: 375 rounds/recruit at \$0.27/round tracer: 10 rounds/recruit at \$0.27/round <u>total</u> : \$104/recruit					\$104
Ammunition Issued to <u>Recruits</u> for Qualification on Squad Assault Weapon (SAW): linked: 287 rounds/recruit at \$0.47/round tracer: 317 rounds/recruit at \$0.47/round <u>total</u> : \$284/recruit					\$284
Ammunition Issued to <u>Training Company Cadre</u> for Instruction on Individual Weapon (M16): ball: 264 rounds/recruit at \$0.27/round tracer: 30 rounds/recruit at \$0.27/round <u>total</u> : \$51/CO/Start				\$80	
Ammunition Issued to <u>Training Company Cadre</u> for Instruction on Squad Assault Weapon (SAW): linked: 0 rounds/recruit at \$0.47/round tracer: 108 rounds/recruit at \$0.47/round total: \$51/CO/Start				\$51	
<b>Subtotal</b>				\$131	\$388

**UTILITIES**

Cost Factor and Computations	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Annual utilities costs of \$309,000 include: electricity (\$156,000); natural gas (\$132,000); water service (\$11,000); sewage service (\$11,000) Assuming 6 starts per year,  $\frac{\$309,000/\text{BN/YR}}{6 \text{ Starts/YR}} = \$51,500/\text{BN/Start}$		\$51,000			
<b>Subtotal</b>		\$51,500			

**SUMMARY**

Cost Factors	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit
Personnel In-Processing:	\$27,000				\$85
Basic Training Support:	\$83,000			\$2166	\$345
Supply Operations:			\$61,000		
Maintenance of Materials:			\$68,000		\$290
Transportation Services:		\$2500	\$56,000	\$1833	
Laundry Services:					\$145
Food Services:					\$788
Personnel Support:			\$26,000		
Ammunition:				\$131	\$388
Utilities:		\$55,500			
<b>GRAND TOTAL:</b>	<b>\$110,000</b>	<b>\$58,000</b>	<b>\$211,000</b>	<b>\$4130</b>	<b>\$2041</b>

**Appendix F:****ADDITIONAL TRAINING PROGRAM COST ESTIMATES**

Cost estimation is an important element of long-range planning and important to the allocation of training resources to support basic combat training. The cost module of the *ARMY* system can support these processes, as well as, provide cost estimates of current training program operations based on realistic training resource utilization. Examples of other types of costs that may be estimated using the *ARMY* system are given below in Tables 9, 10, and 11. The cost estimates are based on training resource scheduling results from Scenario 11.

Tables 9 and 10 summarize annual costs for 1989 and 1990, respectively. Table 11 features additional cost estimates and compares the results from the two years of the planning horizon by annual change and the percent change.

1989 Training Costs	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit	TOTAL
Personnel In-Processing	\$621,000				\$10,403,823	\$11,024,823
Basic Training Support	\$1,909,000			\$1,416,564	\$42,227,282	\$45,552,846
Supply Operations			\$7,015,000			\$7,015,000
Maintenance of Materials			\$7,820,000		\$35,495,397	\$43,315,397
Transportation		\$327,000	\$6,440,000	\$1,198,782		\$7,965,782
Laundry					\$17,747,698	\$17,747,698
Food Service					\$96,449,560	\$96,449,560
Personnel Support			\$2,990,000			\$2,990,000
Ammunition				\$85,674	\$47,490,393	\$47,576,067
Utilities		\$7,259,400				\$7,259,400
<b>TOTAL</b>	<b>\$2,530,000</b>	<b>\$7,586,400</b>	<b>\$24,265,000</b>	<b>\$2,701,020</b>	<b>\$249,814,153</b>	<b>\$286,896,573</b>

Table 9. Summary of 1989 Costs for Scenario 11

1990 Training Costs	\$/bn/yr	\$/bn/start	\$/co/yr	\$/co/start	\$/recruit	TOTAL
Personnel In-Processing	\$540,000				\$9,791,834	\$10,331,834
Basic Training Support	\$1,660,000			\$1,290,936	\$39,743,324	\$42,694,260
Supply Operations			\$6,100,000			\$6,100,000
Maintenance of Materials			\$6,800,000		\$33,407,432	\$40,207,432
Transportation		\$298,000	\$5,600,000	\$1,092,468		\$6,990,468
Laundry					\$16,703,716	\$16,703,716
Food Service					\$90,776,057	\$90,776,057
Personnel Support			\$2,600,000			\$2,600,000
Ammunition				\$78,076	\$44,696,840	\$44,774,916
Utilities		\$6,615,600				\$6,615,600
<b>TOTAL</b>	<b>\$2,200,000</b>	<b>\$6,913,600</b>	<b>\$21,100,000</b>	<b>\$2,461,480</b>	<b>\$235,119,203</b>	<b>\$267,794,283</b>

Table 10. Summary of 1990 Costs for Scenario 11

<b>Cost Summary</b>	<b>1989</b>	<b>1990</b>	<b>Difference</b>	<b>% Change</b>
<b>Total Cost</b>	\$286,896,573	\$267,794,283	(\$19,102,290)	-6.66%
<b>Total Fixed Cost (BN, CO)</b>	\$26,795,000	\$23,300,000	(\$3,495,000)	-13.04%
<b>Total Variable Cost (BN, CO, Recruit)</b>	\$260,101,573	\$244,494,283	(\$15,607,290)	-6.00%
<b>Total Cost by Resource</b>				
Personnel In-Processing	\$11,024,823	\$10,331,834	(\$692,990)	-6.29%
Basic Training Support	\$45,552,846	\$42,694,260	(\$2,858,586)	-6.28%
Supply Operations	\$7,015,000	\$6,100,000	(\$915,000)	-13.04%
Maintenance of Materials	\$43,315,397	\$40,207,432	(\$3,107,965)	-7.18%
Transportation	\$7,965,782	\$6,990,468	(\$975,314)	-12.24%
Laundry	\$17,747,698	\$16,703,716	(\$1,043,982)	-5.88%
Food Service	\$96,449,560	\$90,776,057	(\$5,673,504)	-5.88%
Personnel Support	\$2,990,000	\$2,600,000	(\$390,000)	-13.04%
Ammunition	\$47,576,067	\$44,774,916	(\$2,801,151)	-5.89%
Utilities	\$7,259,400	\$6,615,600	(\$643,800)	-8.87%
<b>Average Variable Cost Per Training Cycle</b>	\$384,745	\$397,135	\$12,390	3.22%
BN Variable Cost per Training Cycle	\$2,040	\$1,946	(\$93)	-4.58%
CO Variable Cost per Training Cycle	\$726	\$693	(\$33)	-4.58%
RECRUIT Variable Cost per Training Cycle	\$381,979	\$394,495	\$12,516	3.28%
<b>Avg. Variable Cost per CO per Cycle by Resource</b>				
Basic Training Support	\$381	\$363	(\$17)	-4.58%
Transportation	\$322	\$308	(\$15)	-4.58%
Ammunition	\$23	\$22	(\$1)	-4.58%
<b>Average Program Cost Per Recruit</b>	\$2,344	\$2,325	(\$19)	-0.82%
<b>Average Cost per Recruit by Resource</b>				
Personnel In-Processing	\$90	\$90	(\$0)	-0.43%
Basic Training Support	\$372	\$371	(\$2)	-0.42%
Supply Operations	\$57	\$53	(\$4)	-7.61%
Maintenance of Materials	\$354	\$349	(\$5)	-1.37%
Transportation	\$65	\$61	(\$4)	-6.76%
Laundry	\$145	\$145	\$0	0.00%
Food Service	\$788	\$788	\$0	0.00%
Personnel Support	\$24	\$23	(\$2)	-7.61%
Ammunition	\$389	\$389	(\$0)	-0.01%
Utilities	\$59	\$57	(\$2)	-3.17%

Table 11. Comparison of Annual Costs for Scenario 11





COMMENTS	PROCEDURES		CODE	COUNTERS
to top of l(t+1) col do routine 21 times fixes rounding pblm for x=250 continue: branch bx  routine to compute l(t+1)  this is the co str used to compute l(t+1) equation for l(t+1) conver to value	Compute l(t+1) for each X(t) needed for the J*(t+1)	y  ALTy1	{goto}testidle0~{down}/re{down 20}~{up} {FOR test1,1,21,1,ALTy1} {let testidle21,testidle21+1} {branch bx}  {down} {left} /mcTESTCOSTR~{bs}~ {right} @INT(+idleco+ra0/lastcostl-ra1/testcostl)~ /v~	test1 22

COMMENTS	PROCEDURES		CODE	COUNTERS
to top of I(t+1) col do routine 21 times when done continue: br lw  if I(t+1)<0, infeas  testidle is a dummy var used to hold the I(t+1) val. Jval1 is where the Jval from J49 will be entered. goto J49 & goto 1st row. test to see if this is the row that is where we get the Jval. if not, test another row for I(t+1). if yes, go right to the col that holds the Jval. put that Jval in Jval1 & return to do the same for the rest of the I(t+1) cases if I(t+1)<0, then infeas br here and do next case	For each pair [x(t), I(t+1)], lookup the J(t+1) value from J*(t+1) table	lx  ALTx1   loop5  getJ  infeas	<pre> goto testidle0~{right}{down}-10~/c~{down 20}~{up}{left} {FOR numcostr1,1,21,1,ALTx1} {branch lw}  {down} {IF @cellpointer("contents")&lt;0}{branch INFEAS} {IF @cellpointer("contents")&gt;20}{branch INFEAS} /mcTESTIDLE~{bs}~ {right} /mcJval1~{bs}~ {goto}JJ49~ {down} {IF @cellpointer("contents")=testidle}{branch GETJ} {branch LOOP5} {right numcostr1} {let Jval1,@cellpointer("contents")} {goto}testidle~  {down}{up} </pre>	<hr/> numcostr1 22 <hr/>

COMMENTS	PROCEDURES	CODE	COUNTERS
top of one-stage cost col enter equation in 1st cell copy eqn to other cells convert eqn to value continue: branch to \w	Compute one-stage cost for each X(t)	\w {goto}cost0~{down}/re{down 20}~{up} {down} +1/st1~ /c~{down 20}~ /rv{down 20}~~ {branch \w}	

COMMENTS	PROCEDURES	CODE	COUNTERS
goto the J(t) col $J(t) = J(t+1) + 1/x(t)$ copy formula to rest of cells convert formula to values continue: branch to \u	Compute J for each X(t)	\v {goto}\jval0~{down}/re{down 20}~{up} {down} @sum(af3..ag3)~ /c~{down 20}~ /rv{down 20}~~ {branch \u}	

COMMENTS	PROCEDURES		CODE	COUNTERS
goto top of J(t) col	Extract max J(t) and corresponding X(t)			
		lw	{goto}jval0~ {branch ALTu1}	
sort thru the J(t) col to find best J value		ALTu1	{down} {IF @cellpointer("contents")=maxjval}{branch GETBEST,J} {branch ALTu1}	
pull out best J & ~X and enter here ->		getbestJ	{let bestjval,@cellpointer("contents")} {left 4} {let bestxval,@cellpointer("contents")} {branch lt}	<u>best J value</u> 0.3189247
continue: branch to lt				<u>best X value</u> 150

COMMENTS	PROCEDURES		CODE	COUNTERS
	Enter best Jval in Jarray and corresponding Xval in Xarray			
enter best j val in currj enter best x val in currxx branch back to lz and do rest of the cells		lt	{let currj,bestjval} {let currxx,bestxval} {goto}lastcostr~ {down idlco+1} {branch LOOP3}	

COMMENTS	PROCEDURES		CODE	COUNTERS
	call in J[t+1]	\c	{goto}D5~/fcdD31..X51~J2.WK4~ {branch \z}	
	print results and continue or quit	\p	{SELECT Z29..AU51} {PRINT selection} {EDIT-COPY C29..X51}{EDIT-PASTE BO10} {SELECT BO10..CJ32}{PRINT selection} {EDIT-CLEAR BO10..CJ32} {FILE-SAVE-ALL} {quit}	

WORKSPACE FOR COMPUTING IDLE TRAINING COMPANIES

co str X(t)	testidle0 I(t+1)	jval1 J(t+1)	1/x(t)	jval0 J(t)
150	13	0.312258	0.006667	0.318925
155	13	0.312043	0.006452	0.318495
160	14	0.312258	0.006250	0.318508
165	15	0.312258	0.006061	0.318319
170	15	0.312043	0.005882	0.317925
175	16	0.312258	0.005714	0.317972
180	16	0.312258	0.005556	0.317814
185	16	0.312043	0.005405	0.317448
190	17	0.312258	0.005263	0.317521
195	17	0.312043	0.005128	0.317171
200	18	0.312258	0.005000	0.317258
205	18	0.312258	0.004878	0.317136
210	18	0.312043	0.004762	0.316805
215	19	0.312258	0.004651	0.316909
220	19	0.312258	0.004545	0.316804
225	19	0.312043	0.004444	0.316487
230	19	0.312043	0.004348	0.316391
235	20	0.312258	0.004255	0.316513
240	20	0.312258	0.004167	0.316425
245	20	0.312043	0.004082	0.316125
250	21	-10.000000	0.004000	-9.996000

max: 0.318925







[illegible]

J <sub>6</sub>	16	x5	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.27876	0.27769	0.27688	0.27591	0.27519	0.27452	0.27346	0.27115	0.27058	0.26899	0.26786	0.26720	0.26597	0.26525	0.26417	0.26257	0.26177	0.26057	0.25937	0.25817	0.25697	0.25577	0.25457
1	0.27876	0.27876	0.27769	0.27688	0.27688	0.27591	0.27519	0.27436	0.27115	0.27058	0.26899	0.26786	0.26720	0.26597	0.26525	0.26417	0.26257	0.26177	0.26057	0.25937	0.25817	0.25697	0.25577
2	0.27876	0.27876	0.27876	0.27769	0.27688	0.27688	0.27591	0.27519	0.27436	0.27115	0.27058	0.26899	0.26786	0.26720	0.26597	0.26525	0.26417	0.26257	0.26177	0.26057	0.25937	0.25817	0.25697
3	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769	0.27688	0.27688	0.27591	0.27519	0.27436	0.27115	0.27058	0.26899	0.26786	0.26720	0.26597	0.26525	0.26417	0.26257	0.26177	0.26057	0.25937
4	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769	0.27688	0.27688	0.27591	0.27519	0.27436	0.27115	0.27058	0.26899	0.26786	0.26720	0.26597	0.26525	0.26417	0.26257	0.26177
5	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769	0.27688	0.27688	0.27591	0.27519	0.27436	0.27115	0.27058	0.26899	0.26786	0.26720	0.26597	0.26476
6	0.28150	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769	0.27688	0.27688	0.27591	0.27519	0.27436	0.27115	0.27058	0.26899	0.26769
7	0.28250	0.28150	0.27983	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769
8	0.28310	0.28250	0.28150	0.28150	0.28150	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769
9	0.28394	0.28310	0.28250	0.28250	0.28150	0.28150	0.28150	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769
10	0.28454	0.28394	0.28310	0.28310	0.28310	0.28250	0.28250	0.28150	0.28150	0.27983	0.27922	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769
11	0.28496	0.28454	0.28394	0.28394	0.28310	0.28310	0.28310	0.28250	0.28250	0.28150	0.28150	0.28150	0.27983	0.27922	0.27922	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27876	0.27769
12	0.28538	0.28496	0.28454	0.28454	0.28394	0.28394	0.28394	0.28310	0.28310	0.28250	0.28250	0.28250	0.28150	0.28150	0.28150	0.27983	0.27922	0.27922	0.27876	0.27876	0.27876	0.27876	0.27769
13	0.28559	0.28538	0.28496	0.28496	0.28454	0.28454	0.28454	0.28394	0.28394	0.28310	0.28310	0.28310	0.28250	0.28250	0.28250	0.28150	0.28150	0.28150	0.28150	0.28150	0.28150	0.27983	0.27893
14	0.28581	0.28559	0.28538	0.28538	0.28496	0.28496	0.28496	0.28454	0.28454	0.28394	0.28394	0.28394	0.28310	0.28310	0.28310	0.28250	0.28250	0.28250	0.28250	0.28250	0.28250	0.28150	0.27983
15	0.28602	0.28581	0.28559	0.28559	0.28538	0.28538	0.28496	0.28496	0.28454	0.28454	0.28394	0.28394	0.28310	0.28310	0.28310	0.28250	0.28250	0.28250	0.28250	0.28250	0.28250	0.28150	0.27983
16	0.28624	0.28602	0.28581	0.28581	0.28559	0.28559	0.28538	0.28538	0.28496	0.28496	0.28454	0.28454	0.28394	0.28394	0.28310	0.28310	0.28310	0.28310	0.28310	0.28310	0.28310	0.28250	0.28150
17	0.28602	0.28624	0.28602	0.28602	0.28581	0.28581	0.28559	0.28559	0.28538	0.28538	0.28496	0.28496	0.28454	0.28454	0.28394	0.28394	0.28310	0.28310	0.28310	0.28310	0.28310	0.28250	0.28150
18	0.28581	0.28602	0.28624	0.28624	0.28602	0.28602	0.28602	0.28581	0.28581	0.28559	0.28559	0.28538	0.28538	0.28496	0.28496	0.28454	0.28454	0.28394	0.28394	0.28310	0.28310	0.28250	0.28150
19	0.28539	0.28581	0.28602	0.28602	0.28624	0.28624	0.28624	0.28602	0.28602	0.28581	0.28581	0.28559	0.28559	0.28538	0.28538	0.28496	0.28496	0.28454	0.28454	0.28394	0.28310	0.28250	0.28150
20	0.28476	0.28517	0.28581	0.28602	0.28624	0.28624	0.28624	0.28602	0.28602	0.28581	0.28581	0.28559	0.28559	0.28538	0.28538	0.28496	0.28496	0.28454	0.28454	0.28394	0.28310	0.28250	0.28150

J7	17	x6	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26232	0.26221	0.26221	0.26171	0.26171	0.26149	0.26117	0.26050	0.26015	0.26015	0.26015	0.26015
1	0.27210	0.27210	0.27124	0.27063	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26232	0.26221	0.26171	0.26149	0.26117	0.26050	0.26015	0.26015	0.26015	0.26117
2	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26232	0.26221	0.26171	0.26149	0.26117	0.26050	0.26015	0.26117
3	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26232	0.26221	0.26171	0.26149	0.26117	0.26050	0.26117
4	0.27256	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26232	0.26221	0.26117
5	0.27463	0.27256	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
6	0.27593	0.27483	0.27358	0.27256	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
7	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
8	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
9	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
10	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
11	0.27871	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
12	0.27892	0.27871	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
13	0.27914	0.27892	0.27871	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286	0.26117
14	0.27935	0.27914	0.27892	0.27871	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386	0.26286
15	0.27957	0.27935	0.27914	0.27892	0.27871	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532	0.26386
16	0.27985	0.27957	0.27935	0.27914	0.27892	0.27871	0.27829	0.27788	0.27727	0.27644	0.27583	0.27483	0.27358	0.27256	0.27210	0.27124	0.27063	0.26985	0.26931	0.26881	0.26790	0.26575	0.26532
17	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957
18	0.27851	0.27894	0.27935	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957	0.27985	0.27914	0.27935	0.27957
19	0.27749	0.27831	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812
20	-9.95479	-9.95397	0.27812	0.27875	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812	0.27875	0.27935	0.27812

[illegible]



J*11	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.24500	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24176	0.24176	0.24063	0.24044	0.23853	0.23853	0.23818	0.23803	0.23788	0.23773	0.23717	0.23664	0.23664	0.23664
1	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24197	0.24176	0.24063	0.23853	0.23818	0.23803	0.23788	0.23773	0.23717	0.23664	0.23664	0.23664
2	0.24543	0.24521	0.24500	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24176	0.24063	0.23853	0.23818	0.23803	0.23788	0.23773	0.23717	0.23664	0.23664
3	0.24569	0.24543	0.24521	0.24521	0.24500	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24176	0.24063	0.23853	0.23818	0.23803	0.23788	0.23773	0.23717	0.23664
4	0.24816	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24176	0.24063	0.23853	0.23818	0.23803
5	0.24916	0.24916	0.24816	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24176	0.24063	0.23853
6	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24176	0.24063	0.23853
7	0.25060	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24197	0.24176	0.24063	0.23853
8	0.25121	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24197	0.24176	0.24063
9	0.25163	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241	0.24197	0.24063
10	0.25204	0.25163	0.25121	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478	0.24328	0.24241
11	0.25226	0.25204	0.25163	0.25163	0.25121	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500	0.24478	0.24478
12	0.25247	0.25226	0.25204	0.25204	0.25163	0.25163	0.25121	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521	0.24521	0.24500
13	0.25269	0.25247	0.25226	0.25226	0.25204	0.25204	0.25163	0.25163	0.25121	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543	0.24543	0.24521
14	0.25290	0.25269	0.25247	0.25247	0.25226	0.25226	0.25204	0.25204	0.25163	0.25163	0.25121	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666	0.24589	0.24543
15	0.25277	0.25290	0.25269	0.25269	0.25247	0.25247	0.25226	0.25226	0.25204	0.25204	0.25163	0.25163	0.25121	0.25121	0.25060	0.25060	0.24977	0.24977	0.24838	0.24816	0.24666
16	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667
17	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667
18	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667
19	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667
20	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667	-9.98667

J\*12

112 / x11

150

155

160

165

170

175

180

185

190

195

200

205

210

215

220

225

230

235

240

245

250

0

1

2

3

4

5

6

7

8

9

10

11

12

13

14

15

16

17

18

19

20

0.23531

0.23596

0.23644

0.23683

0.23737

0.23776

0.23822

0.23876

0.23944

0.24011

0.24077

0.24143

0.24209

0.24275

0.24341

0.24407

0.24473

0.24539

0.24605

0.24671

0.24737

0.24803

0.24869

0.24935

0.25001

0.25067

0.25133

0.25199

0.25265

0.25331

0.25397

0.25463

0.25529

0.25595

0.25661

0.25727

0.25793

0.25859

0.25925

0.25991

0.26057

0.26123

0.26189

0.26255

0.26321

0.26387

0.26453

0.26519

0.26585

0.26651

0.26717

0.26783

0.26849

0.26915

0.26981

0.27047

0.27113

0.27179

0.27245

0.27311

0.27377

0.27443

0.27509

0.27575

0.27641

0.27707

0.27773

0.27839

0.27905

0.27971

0.28037

0.28103

0.28169

0.28235

0.28301

0.28367

0.28433

0.28499

0.28565

0.28631

0.28697

0.28763

0.28829

0.28895

0.28961

0.29027

0.29093

0.29159

0.29225

0.29291

0.29357

0.29423

0.29489

0.29555

0.29621

0.29687

0.29753

0.29819

0.29885

0.29951

0.30017

0.30083

0.30149

0.30215

0.30281

0.30347

0.30413

0.30479

0.30545

0.30611

0.30677

0.30743

0.30809

0.30875

0.30941

0.31007

0.31073

0.31139

0.31205

0.31271

0.31337

0.31403

0.31469

0.31535

0.31601

0.31667

0.31733

0.31799

0.31865

0.31931

0.31997

0.32063

0.32129

0.32195

0.32261

0.32327

0.32393

0.32459

0.32525











J*19	119	118	117	116	115	150	151	152	153	154	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.19362	0.19273	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	0.19134	
1	0.19589	0.19383	0.19351	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	0.19251	
2	0.19699	0.19611	0.19472	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	0.19362	
3	0.19750	0.19689	0.19689	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	0.19589	
4	0.19833	0.19755	0.19750	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	0.19689	
5	0.19894	0.19833	0.19833	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	0.19750	
6	0.19935	0.19894	0.19894	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	0.19833	
7	0.19957	0.19935	0.19935	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	0.19894	
8	0.19978	0.19957	0.19957	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	0.19935	
9	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	
10	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	
11	0.20000	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	
12	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
13	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
14	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
15	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
16	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
17	0.19978	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
18	0.19957	0.19978	0.19978	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	0.20000	
19	0.19894	0.19957	0.19957	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	0.19978	
20	-9.93376	0.19894	0.19935	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	0.19957	

J*20	120	119	118	117	116	150	155	155	155	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.19467	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	
1	0.18684	0.18606	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	
2	0.18806	0.18717	0.18684	0.18531	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	0.18457	
3	0.19023	0.18944	0.18795	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	
4	0.19083	0.19023	0.19023	0.18969	0.18795	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	0.18684	
5	0.19167	0.19088	0.19083	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	0.19023	
6	0.19229	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	0.19167	
7	0.19269	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	
8	0.19290	0.19269	0.19269	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	0.19227	
9	0.19312	0.19290	0.19290	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	0.19269	
10	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	
11	0.19333	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	0.19312	
12	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333	0.19333</													

J*21	121	120	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.17790	0.17790	0.17790	0.17790	0.17769	0.17727	0.17708	0.17708	0.17670	0.17613	0.17613	0.17559	0.17487	0.17419	0.17275	0.17163	0.17154	0.17142	0.16881	0.16858	0.16787	0.16720	0.16720
1	0.18018	0.17822	0.17790	0.17790	0.17790	0.17769	0.17769	0.17769	0.17727	0.17708	0.17708	0.17670	0.17613	0.17559	0.17487	0.17419	0.17275	0.17154	0.17142	0.17142	0.17142	0.17142	0.16881
2	0.18128	0.18018	0.18018	0.17790	0.17790	0.17790	0.17790	0.17790	0.17769	0.17769	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487	0.17275	0.17154	0.17142	0.17142	0.17142	0.17142
3	0.18356	0.18160	0.18128	0.18018	0.18018	0.18018	0.18018	0.18018	0.17790	0.17790	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487	0.17275	0.17154	0.17142	0.17142	0.17142
4	0.18417	0.18356	0.18356	0.18128	0.18128	0.18128	0.18128	0.18128	0.18018	0.18018	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487	0.17275	0.17154	0.17142	0.17142
5	0.18500	0.18417	0.18417	0.18356	0.18356	0.18356	0.18356	0.18356	0.18128	0.18128	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487	0.17275	0.17154	0.17142
6	0.18560	0.18500	0.18500	0.18417	0.18417	0.18417	0.18417	0.18417	0.18356	0.18356	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487	0.17275	0.17142
7	0.18602	0.18560	0.18560	0.18500	0.18500	0.18500	0.18500	0.18500	0.18417	0.18417	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487	0.17275
8	0.18624	0.18602	0.18602	0.18560	0.18560	0.18560	0.18560	0.18560	0.18500	0.18500	0.18500	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559	0.17487
9	0.18645	0.18624	0.18624	0.18602	0.18602	0.18602	0.18602	0.18602	0.18560	0.18560	0.18560	0.18500	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613	0.17559
10	0.18645	0.18645	0.18645	0.18624	0.18624	0.18624	0.18624	0.18624	0.18602	0.18602	0.18602	0.18560	0.18500	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613
11	0.18667	0.18645	0.18645	0.18624	0.18624	0.18624	0.18624	0.18624	0.18602	0.18602	0.18602	0.18560	0.18500	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613
12	0.18667	0.18667	0.18667	0.18645	0.18645	0.18645	0.18645	0.18645	0.18624	0.18624	0.18624	0.18560	0.18500	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613
13	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18645	0.18645	0.18645	0.18560	0.18500	0.18417	0.18356	0.18128	0.18018	0.17790	0.17769	0.17727	0.17708	0.17670	0.17613
14	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667
15	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667
16	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667
17	0.18645	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667
18	0.18624	0.18645	0.18645	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667
19	0.18497	0.18624	0.18624	0.18645	0.18645	0.18645	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667
20	-9.94710	0.18497	0.18545	0.18624	0.18624	0.18624	0.18645	0.18645	0.18645	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667	0.18667

J*22	122	121	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.17124	0.17124	0.17124	0.17102	0.17102	0.17102	0.17082	0.17041	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405	0.16332	0.16332	0.16275	0.16252	0.16252	0.16156	0.15965	0.15920
1	0.17351	0.17124	0.17124	0.17124	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405	0.16332	0.16332	0.16275	0.16252	0.16252	0.16156	0.15965	0.15920
2	0.17462	0.17351	0.17309	0.17124	0.17124	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405	0.16332	0.16332	0.16275	0.16252	0.16252	0.16156	0.15965
3	0.17689	0.17462	0.17420	0.17351	0.17309	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405	0.16332	0.16332	0.16275	0.16252	0.16252	0.16156	0.15965
4	0.17750	0.17689	0.17648	0.17462	0.17420	0.17351	0.17351	0.17309	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405	0.16332	0.16332	0.16275	0.16252
5	0.17833	0.17750	0.17708	0.17689	0.17648	0.17462	0.17462	0.17351	0.17309	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405	0.16332	0.16332	0.16275
6	0.17894	0.17833	0.17792	0.17750	0.17708	0.17689	0.17689	0.17648	0.17462	0.17462	0.17351	0.17309	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763	0.16654	0.16554	0.16405
7	0.17935	0.17894	0.17852	0.17833	0.17792	0.17750	0.17750	0.17708	0.17689	0.17689	0.17648	0.17462	0.17462	0.17351	0.17309	0.17124	0.17124	0.17102	0.17082	0.17003	0.16946	0.16892	0.16763
8	0.17957	0.17935	0.17894	0.17894	0.17852	0.17833	0.17833	0.17792	0.17750	0.17750	0.17708	0.17689	0.17689	0.17648	0.17462	0.17462	0.17351	0.17309	0.17124	0.17124	0.17102	0.17082	0.17003
9	0.17978	0.17957	0.17935	0.17935	0.17894	0.17894	0.17894	0.17852	0.17833	0.17833	0.17792	0.17750	0.17750	0.17708	0.17689	0.17689	0.17648	0.17462	0.17462	0.17351	0.17309	0.17124	0.17124
10	0.17978	0.17978	0.17957	0.17957	0.17935	0.17935	0.17935	0.17894	0.17894	0.17894	0.17852	0.17833	0.17833	0.17792	0.17750	0.17750	0.17708	0.17689	0.17689	0.17648	0.17462	0.17462	0.17351
11	0.18000	0.17978	0.17978	0.17978	0.17957	0.17957	0.17957	0.17935	0.17935	0.17935	0.17894	0.17894	0.17894	0.17852	0.17833	0.17833	0.17792	0.17750	0.17750	0.17708	0.17689	0.17689	0.17648
12	0.18000	0.18000	0.17978	0.17978	0.17978	0.17978	0.17978	0.17957	0.17957	0.17957	0.17935	0.17935	0.17935	0.17894	0.17894	0.17894	0.17852	0.17833	0.17833	0.17792	0.17750	0.17750	0.17708
13	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.17978	0.17978	0.17978	0.17957	0.17957	0.17957	0.17935	0.17935	0.17935	0.17894	0.17894	0.17894	0.17852
14	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000
15	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000
16	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000
17	0.17978	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000
18	0.17957	0.17978	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000
19	0.17757	0.17900	0.17978	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000
20	-9.95357	-9.95355	0.17872	0.17900	0.17978	0.17978	0.17978	0.17978	0.17978	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000	0.18000				

[illegible]

J*24	124	123	122	121	120	119	118	117	116	115	114	113	112	111	110	109	108	107	106	105	104	103	102	101	100	99	98	97	96	95	94	93	92	91	90	89	88	87	86	85	84	83	82	81	80	79	78	77	76	75	74	73	72	71	70	69	68	67	66	65	64	63	62	61	60	59	58	57	56	55	54	53	52	51	50	49	48	47	46	45	44	43	42	41	40	39	38	37	36	35	34	33	32	31	30	29	28	27	26	25	24	23	22	21	20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																																											
	0.15790	0.15790	0.15790	0.15769	0.15749	0.15651	0.15572	0.15352	0.15318	0.15286	0.15213	0.15044	0.14936	0.14909	0.14832	0.14791	0.14791	0.14791	0.14751	0.14713	0.14683	0.14632	0.14583	0.14533	0.14483	0.14433	0.14383	0.14333	0.14283	0.14233	0.14183	0.14133	0.14083	0.14033	0.13983	0.13933	0.13883	0.13833	0.13783	0.13733	0.13683	0.13633	0.13583	0.13533	0.13483	0.13433	0.13383	0.13333	0.13283	0.13233	0.13183	0.13133	0.13083	0.13033	0.12983	0.12933	0.12883	0.12833	0.12783	0.12733	0.12683	0.12633	0.12583	0.12533	0.12483	0.12433	0.12383	0.12333	0.12283	0.12233	0.12183	0.12133	0.12083	0.12033	0.11983	0.11933	0.11883	0.11833	0.11783	0.11733	0.11683	0.11633	0.11583	0.11533	0.11483	0.11433	0.11383	0.11333	0.11283	0.11233	0.11183	0.11133	0.11083	0.11033	0.10983	0.10933	0.10883	0.10833	0.10783	0.10733	0.10683	0.10633	0.10583	0.10533	0.10483	0.10433	0.10383	0.10333	0.10283	0.10233	0.10183	0.10133	0.10083	0.10033	0.09983	0.09933	0.09883	0.09833	0.09783	0.09733	0.09683	0.09633	0.09583	0.09533	0.09483	0.09433	0.09383	0.09333	0.09283	0.09233	0.09183	0.09133	0.09083	0.09033	0.08983	0.08933	0.08883	0.08833	0.08783	0.08733	0.08683	0.08633	0.08583	0.08533	0.08483	0.08433	0.08383	0.08333	0.08283	0.08233	0.08183	0.08133	0.08083	0.08033	0.07983	0.07933	0.07883	0.07833	0.07783	0.07733	0.07683	0.07633	0.07583	0.07533	0.07483	0.07433	0.07383	0.07333	0.07283	0.07233	0.07183	0.07133	0.07083	0.07033	0.06983	0.06933	0.06883	0.06833	0.06783	0.06733	0.06683	0.06633	0.06583	0.06533	0.06483	0.06433	0.06383	0.06333	0.06283	0.06233	0.06183	0.06133	0.06083	0.06033	0.05983	0.05933	0.05883	0.05833	0.05783	0.05733	0.05683	0.05633	0.05583	0.05533	0.05483	0.05433	0.05383	0.05333	0.05283	0.05233	0.05183	0.05133	0.05083	0.05033	0.04983	0.04933	0.04883	0.04833	0.04783	0.04733	0.04683	0.04633	0.04583	0.04533	0.04483	0.04433	0.04383	0.04333	0.04283	0.04233	0.04183	0.04133	0.04083	0.04033	0.03983	0.03933	0.03883	0.03833	0.03783	0.03733	0.03683	0.03633	0.03583	0.03533	0.03483	0.03433	0.03383	0.03333	0.03283	0.03233	0.03183	0.03133	0.03083	0.03033	0.02983	0.02933	0.02883	0.02833	0.02783	0.02733	0.02683	0.02633	0.02583	0.02533	0.02483	0.02433	0.02383	0.02333	0.02283	0.02233	0.02183	0.02133	0.02083	0.02033	0.01983	0.01933	0.01883	0.01833	0.01783	0.01733	0.01683	0.01633	0.01583	0.01533	0.01483	0.01433	0.01383	0.01333	0.01283	0.01233	0.01183	0.01133	0.01083	0.01033	0.00983	0.00933	0.00883	0.00833	0.00783	0.00733	0.00683	0.00633	0.00583	0.00533	0.00483	0.00433	0.00383	0.00333	0.00283	0.00233	0.00183	0.00133	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.00033	0.00083	0.0

J <sub>25</sub> 125° x 24°	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.15124	0.15124	0.15124	0.15045	0.14984	0.14780	0.14763	0.14746	0.14687	0.14531	0.14436	0.14421	0.14356	0.14325	0.14296	0.14269	0.14258	0.14258	0.14211	0.14166	0.14124
1	0.15351	0.15145	0.15124	0.15124	0.15124	0.15045	0.14984	0.14780	0.14763	0.14763	0.14746	0.14687	0.14531	0.14436	0.14421	0.14356	0.14356	0.14325	0.14296	0.14259	0.14269
2	0.15462	0.15370	0.15351	0.15145	0.15124	0.15124	0.15045	0.14984	0.14984	0.14780	0.14780	0.14763	0.14763	0.14746	0.14687	0.14687	0.14687	0.14436	0.14421	0.14356	0.14356
3	0.15699	0.15483	0.15462	0.15370	0.15351	0.15145	0.15124	0.15124	0.15045	0.15045	0.14984	0.14984	0.14984	0.14780	0.14780	0.14763	0.14746	0.14746	0.14687	0.14687	0.14531
4	0.15750	0.15708	0.15689	0.15483	0.15462	0.15370	0.15351	0.15145	0.15124	0.15124	0.15124	0.15124	0.15045	0.15045	0.14984	0.14984	0.14780	0.14780	0.14763	0.14763	0.14763
5	0.15833	0.15750	0.15750	0.15708	0.15689	0.15483	0.15462	0.15370	0.15351	0.15309	0.15145	0.15124	0.15124	0.15124	0.15124	0.15124	0.15045	0.15045	0.14984	0.14984	0.14884
6	0.15894	0.15852	0.15833	0.15750	0.15750	0.15708	0.15689	0.15483	0.15462	0.15423	0.15370	0.15351	0.15309	0.15145	0.15124	0.15124	0.15124	0.15124	0.15124	0.15124	0.15124
7	0.15935	0.15894	0.15894	0.15852	0.15833	0.15750	0.15708	0.15689	0.15689	0.15689	0.15483	0.15462	0.15423	0.15370	0.15351	0.15351	0.15309	0.15309	0.15309	0.15309	0.15309
8	0.15957	0.15935	0.15935	0.15894	0.15894	0.15852	0.15833	0.15750	0.15750	0.15708	0.15708	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689
9	0.15978	0.15957	0.15957	0.15935	0.15935	0.15894	0.15852	0.15833	0.15792	0.15792	0.15750	0.15750	0.15708	0.15708	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689	0.15689
10	0.15978	0.15978	0.15978	0.15957	0.15957	0.15935	0.15894	0.15894	0.15852	0.15852	0.15852	0.15833	0.15792	0.15750	0.15750	0.15750	0.15708	0.15708	0.15689	0.15689	0.15689
11	0.16000	0.15978	0.15978	0.15978	0.15978	0.15957	0.15935	0.15894	0.15894	0.15894	0.15894	0.15894	0.15852	0.15852	0.15833	0.15833	0.15750	0.15750	0.15750	0.15750	0.15708
12	0.16000	0.16000	0.16000	0.15978	0.15978	0.15978	0.15978	0.15957	0.15957	0.15935	0.15935	0.15935	0.15894	0.15894	0.15894	0.15894	0.15852	0.15852	0.15833	0.15833	0.15792
13	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.15978	0.15978	0.15978	0.15957	0.15957	0.15957	0.15935	0.15935	0.15935	0.15935	0.15894	0.15894	0.15894	0.15894	0.15852
14	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.15978	0.15978	0.15978	0.15978	0.15978	0.15957	0.15957	0.15957	0.15957	0.15935	0.15935	0.15935	0.15935	0.15894
15	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.15978	0.15978	0.15978	0.15978	0.15957	0.15957	0.15957	0.15935	0.15935	0.15957	0.15957	0.15935
16	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.15978	0.15978	0.15978	0.15978	0.15978	0.15978	0.15978	0.15978
17	0.15894	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.15978
18	-9.97333	0.15800	0.15894	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.15978
19	-9.98000	-9.97333	-9.97333	0.15800	0.15894	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000
20	-9.98667	-9.98000	-9.98000	-9.97333	0.15800	0.15894	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000	0.16000

[illegible]



J*29	129 / x28	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.11904	0.11904	0.11882	0.11862	0.11868	0.11668	0.11668	0.11668	0.11668	0.11626	0.11567	0.11567	0.11376	0.11359	0.11359	0.11161	0.11044	0.10888	0.10888	0.10804	0.10678	0.10559
1	0.11929	0.11904	0.11904	0.11882	0.11793	0.11668	0.11668	0.11668	0.11668	0.11626	0.11567	0.11567	0.11376	0.11359	0.11359	0.11161	0.11044	0.10888	0.10888	0.10804	0.10678	0.10559
2	0.12196	0.12174	0.12039	0.12039	0.11904	0.11904	0.11904	0.11904	0.11904	0.11882	0.11862	0.11862	0.11793	0.11668	0.11668	0.11567	0.11567	0.11376	0.11359	0.11359	0.11161	0.11044
3	0.12421	0.12402	0.12441	0.12441	0.12402	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399	0.12399
4	0.12521	0.12500	0.12441	0.12441	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402	0.12402
5	0.12746	0.12727	0.12571	0.12571	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500	0.12500
6	0.12860	0.12838	0.12767	0.12767	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727	0.12727
7	0.13085	0.13066	0.12909	0.12909	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838	0.12838
8	0.13228	0.13210	0.13146	0.13146	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105	0.13105
9	0.13370	0.13352	0.13288	0.13288	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249	0.13249
10	0.13512	0.13494	0.13430	0.13430	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391	0.13391
11	0.13654	0.13636	0.13572	0.13572	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533	0.13533
12	0.13796	0.13778	0.13714	0.13714	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675	0.13675
13	0.13938	0.13920	0.13856	0.13856	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817	0.13817
14	0.14080	0.14062	0.13998	0.13998	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959	0.13959
15	0.14222	0.14204	0.14142	0.14142	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103	0.14103
16	0.14364	0.14346	0.14284	0.14284	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245	0.14245
17	0.14506	0.14488	0.14426	0.14426	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387	0.14387
18	0.14648	0.14630	0.14568	0.14568	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529	0.14529
19	0.14790	0.14772	0.14710	0.14710	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671	0.14671
20	0.14932	0.14914	0.14852	0.14852	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813	0.14813

J\*30

130 / x29

150

155

160

165

170

175

180

185

190

195

200

205

210

215

220

225

230

235

240

245

250

J*31	131	130	150	155	160	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	0.10571	0.10356	0.10356	0.10336	0.10182	0.10018	0.09933	0.09806	0.09752	0.09533	0.09533	0.09429	0.09429	0.09417	0.09351	0.09289	0.09230	0.09180	0.09190	0.09160	0.09135	0.09135
1	0.10571	0.10571	0.10571	0.10571	0.10356	0.10356	0.10336	0.10182	0.10018	0.10018	0.10018	0.09933	0.09806	0.09752	0.09752	0.09533	0.09429	0.09429	0.09417	0.09351	0.09351	0.09289
2	0.10591	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10356	0.10356	0.10336	0.10336	0.10182	0.10182	0.10018	0.09933	0.09806	0.09806	0.09752	0.09752	0.09533	0.09533
3	0.10863	0.10863	0.10726	0.10591	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10356	0.10356	0.10336	0.10336	0.10182	0.10182	0.10018	0.10018	0.09833
4	0.11129	0.11069	0.10863	0.10863	0.10726	0.10726	0.10669	0.10591	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10356	0.10356	0.10356	0.10356	0.10336
5	0.11188	0.11188	0.11129	0.11129	0.11069	0.10863	0.10863	0.10863	0.10726	0.10669	0.10591	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571	0.10571
6	0.11455	0.11394	0.11208	0.11188	0.11188	0.11129	0.11129	0.11129	0.10963	0.10863	0.10863	0.10863	0.10863	0.10726	0.10591	0.10591	0.10591	0.10571	0.10571	0.10571	0.10571	0.10571
7	0.11526	0.11526	0.11455	0.11455	0.11394	0.11208	0.11188	0.11188	0.11129	0.11129	0.11129	0.11129	0.11129	0.11069	0.10863	0.10863	0.10863	0.10863	0.10863	0.10863	0.10863	0.10863
8	0.11793	0.11732	0.11591	0.11526	0.11526	0.11455	0.11455	0.11455	0.11288	0.11188	0.11188	0.11188	0.11188	0.11129	0.11129	0.11129	0.11129	0.11129	0.11129	0.11129	0.11129	0.11129
9	0.11835	0.11793	0.11793	0.11793	0.11732	0.11591	0.11526	0.11526	0.11455	0.11455	0.11455	0.11455	0.11455	0.11394	0.11208	0.11188	0.11188	0.11188	0.11188	0.11188	0.11188	0.11188
10	0.11937	0.11876	0.11835	0.11835	0.11793	0.11793	0.11793	0.11793	0.11626	0.11526	0.11526	0.11526	0.11526	0.11455	0.11455	0.11455	0.11455	0.11455	0.11455	0.11455	0.11455	0.11455
11	0.11937	0.11937	0.11937	0.11937	0.11876	0.11835	0.11835	0.11835	0.11793	0.11793	0.11793	0.11793	0.11793	0.11732	0.11591	0.11526	0.11526	0.11526	0.11526	0.11526	0.11526	0.11526
12	0.11978	0.11978	0.11978	0.11978	0.11937	0.11937	0.11937	0.11937	0.11835	0.11835	0.11835	0.11835	0.11835	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793
13	0.12000	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11937	0.11937	0.11937	0.11937	0.11937	0.11876	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793	0.11793
14	0.12000	0.12000	0.12000	0.12000	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11937	0.11937	0.11937	0.11937	0.11937	0.11937	0.11937	0.11937	0.11937
15	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978	0.11978
16	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000
17	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000
18	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000
19	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000
20	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000	0.12000

[illegible]











[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

[illegible][illegible]





[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

X*9	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
19/x/8																					
0	150	155	160	165	170	175	180	185	190	195	200	205	210	210	215	220	225	230	235	240	250
1	150	150	155	160	165	170	175	180	185	185	190	195	200	205	205	210	215	220	225	230	235
2	150	150	150	150	155	160	160	165	170	175	180	185	190	190	190	195	200	200	205	210	215
3	150	150	160	150	150	150	155	160	160	165	170	170	175	180	180	185	190	190	195	200	205
4	150	150	160	160	150	150	150	155	155	160	165	165	170	170	175	180	185	180	185	190	195
5	150	150	160	160	160	150	160	150	150	150	155	155	160	160	165	165	170	170	175	180	185
6	150	150	160	160	160	150	160	150	160	150	150	150	155	155	160	165	160	165	165	170	175
7	150	150	160	160	160	150	160	150	160	150	160	150	150	150	150	155	155	155	160	160	165
8	150	150	160	160	160	150	160	150	160	150	160	150	150	160	150	150	150	150	150	150	155
9	150	150	150	150	160	150	160	150	160	150	160	150	150	160	150	150	160	150	150	150	160
10	150	150	150	150	150	150	160	150	160	150	160	150	150	160	150	150	160	150	150	150	160
11	150	150	150	150	150	150	150	150	160	150	160	150	150	160	150	150	160	150	150	150	160
12	150	150	150	150	150	150	150	150	150	150	160	150	150	160	150	150	160	150	150	150	160
13	150	150	150	150	150	150	150	150	150	150	150	150	150	160	150	150	160	150	150	150	160
14	155	150	160	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	160
15	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	160
16	155	160	150	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	160
17	160	160	175	155	150	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	160
18	155	150	175	155	175	155	150	155	150	150	150	150	150	150	150	150	150	150	150	150	160
19	150	160	150	150	175	155	175	155	150	150	150	150	150	150	150	150	150	150	150	150	160
20	155	150	175	155	150	150	175	155	175	160	150	150	155	150	150	150	150	150	150	150	160

[illegible]

[illegible][illegible]

X*13 113X12	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
1	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
2	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
3	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
4	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
5	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
6	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
7	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
8	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
9	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
10	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
11	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
12	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
13	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
14	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
15	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
16	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
17	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
18	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
19	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
20	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250

X*14	X*14	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
114x13	0	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	250	250	250	250	250
	1	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250	255
	2	150	150	150	155	160	165	165	170	175	175	180	185	185	190	195	200	200	200	205	210	210
	3	155	160	150	150	150	155	160	165	160	165	165	170	170	175	180	180	185	185	190	190	190
	4	155	160	150	155	160	150	150	150	150	150	155	155	160	160	165	165	170	170	170	175	175
	5	155	150	150	155	160	150	155	160	160	150	150	155	150	150	150	150	155	160	160	160	165
	6	155	170	150	155	150	150	155	160	160	150	155	155	160	160	150	150	150	150	150	150	150
	7	150	150	150	155	170	150	155	150	150	150	155	155	160	160	150	150	155	160	160	160	160
	8	150	150	150	150	150	150	155	200	160	150	155	155	150	150	150	155	155	160	160	160	160
	9	150	150	150	150	150	150	150	150	150	150	155	155	170	160	150	155	155	150	150	150	150
	10	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	155	155	200	170	160	150
	11	155	170	150	155	150	150	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150
	12	155	150	150	155	170	150	155	150	150	150	150	155	150	150	150	150	150	150	150	150	150
	13	155	150	150	150	155	150	155	200	160	150	155	155	150	150	150	150	150	150	150	150	150
	14	155	150	150	150	155	150	155	150	150	150	155	155	170	160	150	155	155	150	150	150	150
	15	155	150	150	155	150	150	155	150	150	150	155	155	150	150	150	155	155	200	170	160	150
	16	155	150	150	150	155	150	155	150	150	150	155	155	150	150	150	155	155	150	150	150	150
	17	155	150	150	155	150	150	155	150	150	150	155	155	150	150	150	155	155	150	150	150	150
	18	150	150	150	150	155	150	155	150	150	150	155	155	150	150	150	155	155	150	150	150	150
	19	150	150	150	150	150	150	155	150	150	150	155	155	150	150	150	155	155	150	150	150	150
	20	150	150	150	150	150	150	150	150	150	150	155	155	150	150	150	155	155	150	150	150	150



[illegible][illegible]



[illegible][illegible]





[illegible][illegible]

[illegible]

X*28 12B/A27	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250															
0	150	150	150	150	150	150	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250									
1	155	175	150	150	150	150	150	150	150	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250						
2	155	150	155	160	250	150	150	150	150	150	150	150	150	150	150	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
3	155	150	155	160	250	155	160	250	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
4	155	150	155	160	250	155	160	250	155	160	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
5	155	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
6	155	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	155	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160
7	155	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	155	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160
8	155	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	155	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160
9	155	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	155	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160
10	155	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	155	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160
11	150	150	155	160	250	155	160	250	155	160	150	155	170	160	150	150	155	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160	160
12	150	150	150	150	155	155	150	155	155	150	150	155	170	150	150	150	155	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
13	150	150	150	150	150	150	150	155	155	150	150	155	170	150	150	150	155	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
14	150	150	150	150	150	150	150	150	150	150	150	155	170	150	150	150	155	155	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
15	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
16	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
17	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
18	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
19	150	160	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
20	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150

X*29 129x28	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	150	150	155	160	150	150	150	155	160	160	160	165	170	170	175	180	185	185	190	250	195
1	150	150	150	150	150	155	160	180	150	150	150	150	155	155	160	160	165	170	170	175	175
2	250	150	150	150	150	150	150	150	155	160	160	180	150	150	150	150	150	155	155	160	160
3	250	150	170	150	150	150	150	150	150	150	150	150	155	155	160	160	230	150	150	150	150
4	160	150	170	150	150	170	175	150	150	150	150	150	150	150	150	150	150	155	155	160	160
5	250	150	155	160	150	170	160	150	170	250	150	150	150	150	150	150	150	150	150	150	150
6	160	150	170	150	150	155	160	150	170	250	150	150	185	170	175	150	150	150	150	150	150
7	250	150	155	160	150	170	160	150	155	160	160	150	185	170	150	150	150	185	170	250	175
8	160	150	170	150	150	155	160	150	170	250	150	150	155	155	160	160	150	185	170	250	150
9	150	150	155	160	150	170	150	150	155	160	160	150	185	170	150	150	150	155	155	160	160
10	160	150	155	160	150	155	160	150	170	250	150	150	155	155	160	160	150	185	170	250	150
11	150	150	155	160	150	155	160	150	155	160	160	150	155	155	160	160	150	185	170	250	150
12	150	150	155	160	150	155	160	150	155	160	160	150	155	155	160	160	150	185	170	250	150
13	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
14	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
15	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
16	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
17	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
18	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
19	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160
20	150	150	150	150	150	150	150	150	155	160	150	150	155	155	160	160	150	155	155	160	160

X*30 130x29	150	155	160	165	170	175	180	185	190	195	200	205	210	215	220	225	230	235	240	245	250
0	155	160	165	170	170	175	180	185	190	195	200	205	210	210	215	220	230	235	240	245	250
1	150	150	150	155	160	160	165	170	175	175	180	185	185	190	195	200	200	205	210	210	215
2	150	150	150	150	150	150	150	155	160	160	165	165	170	175	175	180	180	185	185	190	190
3	150	150	150	150	150	150	150	150	150	150	150	155	155	160	160	165	165	170	170	175	175
4	155	160	250	150	150	150	150	150	150	150	150	150	150	150	150	150	150	155	155	160	160
5	165	210	150	155	170	160	150	150	150	150	150	150	150	150	150	150	150	150	150	150	150
6	155	150	150	155	170	150	150	155	160	160	250	150	150	150	150	150	150	150	150	150	150
7	165	210	150	155	150	150	150	155	210	150	150	155	155	160	160	250	150	150	150	150	150
8	155	150	150	155	170	150	150	155	150	150	150	165	155	210	150	150	150	155	155	160	160
9	165	160	150	155	150	150	150	155	210	150	150	165	155	150	150	150	150	155	155	210	150
10	150	150	150	155	170	150	150	155	150	150	150	165	155	210	150	150	150	155	155	150	150
11	165	150	150	150	150	150	150	155	160	160	150	155	155	150	150	150	150	155	155	210	150
12	150	150	150	155	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
13	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
14	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
15	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
16	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
17	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
18	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
19	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150
20	150	150	150	150	150	150	150	155	150	150	150	165	155	160	160	150	150	155	155	150	150

[illegible][illegible]

[illegible][illegible]



[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

[illegible][illegible]

## REFERENCES

1. Afentakis, P., March 1987, A Parallel Heuristic Algorithm for Lot-Sizing in Multistage Production Systems, *IIE Transactions*, pp. 34-42.
2. Afentakis, P., Gavish, B. and Karmarkar, U., 1984, Computationally Efficient Optimal Solutions to the Lot-Sizing Problem in Multistage Assembly Systems, *Management Science*, VOL. 30, pp. 222-239.
3. Bertsekas, D., 1987, *Dynamic Programming*, Prentice-Hall, Englewood Cliffs, NJ.
4. Billington, P., McClain, J., and Thomas, L., 1986, Heuristics for Multilevel Lot-Sizing with a Bottleneck, *Management Science*, VOL. 32, pp. 989-1006.
5. Blackburn, J. and Millen, R., 1982, Improved Heuristics for Multi-Stage Requirements Planning Systems, *Management Science*, VOL. 28, pp. 44-56.
6. Crowston, W., Wagner, M., and Williams, J., 1973, Economic Lot Size Determination in Multi-Stage Assembly Systems, *Management Science*, VOL. 19, pp. 517-527.
7. Florian, M. and Klein, M., 1971, Deterministic Production Planning with Concave Costs and Capacity Constraints, *Management Science*, VOL. 18, pp. 12-20.
8. Graves, S., 1981, Multi-Stage Lot-Sizing: An Iterative Procedure, *Management Science*, VOL. 16, pp. 95-110.
9. Manne, A., 1958, Programming of Economic Lot Sizes, *Management Science*, VOL. 4, pp. 115-135.
10. McGinnis, M., 1989, *Training Base Management Model User's Guide*, Version 2.0, U.S. Army Training and Doctrine Command Headquarters, Fort Monroe, VA.



11. Rao, Poornachandra P., 1990, A Dynamic Programming Approach to Determine Optimal Manpower Recruitment Policies, *Operations Research*, VOL. 41, pp. 983-988.
12. Wagner, M. and Whitin, T., 1958, Dynamic Version of the Economic Lot Size Model, *Management Science*, VOL. 5, pp. 89-96.
13. Yang, T. and Ignizio, J., 1987, Algorithm for the Scheduling of Army Battalion Exercises, *Computers and Operations Research*, VOL. 14, pp. 479-491.
14. Zangwill, W., 1966, A Deterministic Multi-Period Production Scheduling Model with Backlogging, *Management Science*, VOL. 13, pp. 105-119.